# On-the-run Premia, Settlement Fails, and Central Bank Access

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# On-the-run Premia, Settlement Fails, and Central Bank Access*

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## Abstract

The premium on "on-the-run" Treasuries (i.e. the most recently issued ones) is an anomaly. I explain it using a model in which primary dealers hold inventories of Treasuries. Primary dealers are more likely to hold large inventories of on-the-run Treasuries. There is also less variation across primary dealers in the available stock of on-the-run Treasuries compared with all other, so-called off-the-run Treasuries. Because on-the-run Treasuries are easier to find, they trade at a premium. My theory is consistent with the USD 40 billion of Treasury contracts that fail to settle each day, with the median failure rate of off-the-run Treasuries being almost twice that of on-the-run Treasuries. I use the model to analyse the effects of granting access to central bank facilities to non-banks active in the Treasury market. Broad access stimulates trading and reduces the on-the-run premium, but settlement fails increase and, counterintuitively, only primary dealers benefit.

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# 1 Introduction

With an average daily trading volume of half a trillion US dollars, the US Treasury market is one of the most important and liquid markets in the US financial system, crucial to the conduct of monetary policy and a key pillar of the US economy. Despite its importance, the US Treasury market exhibits some irregularities, which I describe in detail in the next section and summarise here.[1]

First, it is well known that on-the-run Treasuries – the most recently issued Treasuries – trade at significantly lower yields, higher prices and lower repo rates than other Treasuries (known as off-the-run) with similar cash flows and maturity dates, giving rise to a puzzling arbitrage opportunity known as the "on-the-run" premium (see Vayanos and Weill (2008), D'Amico and Pancost (2022) and figure 2a in the next section).[2] Second, despite trading at a premium, the volume of trades in on-the-run Treasuries is much larger than that in off-the-run Treasuries (see figure 2b in the next section). Third, on average USD 40 billion of Treasury contracts fail to settle each day (see figure 3a in the next section). Fourth, interestingly, failure rates differ by Treasury type, with on-the-run Treasuries having a median settlement failure rate almost half that of off-the-run Treasuries (see figure 3b in the next section).

These stylised facts and irregularities raise the following questions: How can there be a premium on certain Treasuries and why is it always on the on-the-run Treasuries? Why do the cheaper off-the-run Treasuries trade at lower volumes? How can there be settlement fails in a benchmark market such as the US Treasury market, and why do the off-the-run Treasuries fail to settle more often?

In the first part of the paper, I develop a model of the US Treasury market to answer these questions. In the second part, which I describe in more detail below, I use it to conduct policy analysis motivated by current discussions about how to restructure the market.

The US Treasury market model incorporates the key features of the market that I describe in section 2: It is an over-the-counter (OTC) market where primary dealers are the first acquirers of Treasuries at the primary auction. I assume that there are three types of agents: sellers, buyers and primary dealers. A seller is any financial entity other than a primary dealer, such as a non-bank, that sells Treasuries short. A buyer is akin to a long-term holder of Treasuries, such as a pension fund. There are different types of Treasuries, on- and off-the-run, and to simplify the model I do not model the primary auction of Treasuries, but assume that primary dealers are endowed with the latest issue of Treasuries.[3] Buyers have the highest valuation for Treasuries, but the market is segmented and they cannot contact primary dealers directly, only through sellers.

In a first market, sellers sell financial contracts to buyers that promise to deliver a specific type of Treasury (for short, on- or off-the-run, including maturity date). Because the seller can fail to settle, the contract is secured by collateral. Next, sellers contact primary dealers to buy the desired type of Treasury in an OTC market. There, the seller is randomly matched with a primary dealer. The primary dealers always have the most recent issue of Treasuries in their inventory, as they have just been auctioned. However, depending on their trading history, they may not have enough of the desired

[1] The average daily trading volume is the volume reported to TRACE between February and October 2023. The TRACE data is available here: https://www.finra.org/finra-data/browse-catalog/about-treasury/monthly-file.

[2] See figure A.4 in the appendix for a graphical representation of the on-the-run cycle.

[3] In the extension in section 10 the auction is included.

off-the-run Treasuries. In this case, the seller fails to settle and he delivers as many Treasuries as possible to the buyer in accordance with their contracts. If necessary, the buyer seizes the collateral to cover the undelivered amount. These fails do not occur with on-the-run Treasuries because all primary dealers hold the same inventories since they were just filled up. Once all the trades have been conducted, the sellers can deposit any remaining idle balances in a central bank facility and receive an interest rate on them.[4] Sellers then go into the next sequence of trades.

In equilibrium, I show that the occurrence of settlement fails leads to a preference for the safer on-the-run Treasuries. Because they have been in the market for a shorter time, sellers are more likely to find them and they have a greater chance to settle. Therefore, on-the-run Treasuries trade at a premium and in greater volume than off-the-run Treasuries with the same cash flow and maturity date, explaining the stylised facts for the US Treasury market mentioned in the beginning. I also provide empirical evidence that lower inventories imply more settlement fails in off-the-run Treasuries, and more such settlement fails imply higher on-the-run premia, as predicted by the model.

In a second part of the paper, I use the model to shed light on the current policy discussion about the need to restructure the Treasury market (see the discussion at the Jackson Hole conference by Duffie (2023)). The background to this discussion is epitomised by the US Treasury market crisis of March 2020. In the aftermath of the great financial crisis of 2007, primary dealers faced tighter regulatory constraints, leading them to reduce their balance sheet space for Treasuries. At the same time, the US Treasury market grew strongly. Non-bank financial institutions have filled the space left by primary dealers.[5] But in March 2020, the presence of non-banks in the US Treasury market led to a rapid drying up of liquidity and a sharp decline in market depth, exacerbated by the reluctance of primary dealers to take more US Treasuries onto their balance sheets (Eren and Wooldridge (2021)). Off-the-run Treasuries were at the epicentre of the crisis (Wells (2023)). This is reflected in the on-the-run premia across all maturities, which rose sharply as shown in figure 1.[6]

Among their ten recommendations for the US Treasury market, the Working Group on Treasury Market Liquidity led by Duffie, Geithner, Parkinson and Stein recommends broad access to central bank repo financing (Duffie et al. (2021)). They criticise the current facility for providing limited access to primary dealers and banks rather than a broad range of market participants.[7]

The model helps to understand the impact of broad access to central bank facilities on prices, premia, traded quantities, fails and profits (in a general setting in normal times). A first observation is that the facility is not a substitute for trading, but rather complements it. In this model, the facility (like a

[4] The facility in my model can be interpreted as a deposit facility or a reverse repo facility where I focus on the cash leg and abstract from the collateral part. First, the facility's repos are general collateral repos and the cash lender is willing to receive any security that falls into a broad class. He does not search for a specific security (Bowman et al. (2017)). Second, even if the facility were to provide a specific security that was sought, the security would have to be returned the next day and the facility would only provide temporary availability.

[5] On the BrokerTec platform, one of the main marketplaces, non-banks, especially principal trading firms, already accounted for more than half of the trading in benchmark 5-year, 10-year and 30-year bonds in 2015. Traditional banks and dealers had a share of 30-40%.

[6] As in Christensen et al. (2017), the on-the-run yield is subtracted from the par yield of seasoned bonds. The data are taken from the FED yield curve, which is an updated version of the original Gürkaynak-Sack-Wright curve (Gürkaynak et al. (2010)), https://www.federalreserve.gov/data/tips-yield-curve-and-inflation-compensation.htm, and the FRB-H15 tables, https://www.federalreserve.gov/releases/h15/.

[7] Another example is the FED's reverse repo facility. An increasing number of institutions already access this facility (Frost et al. (2015), Baklanova et al. (2015) and Marte (2021)).

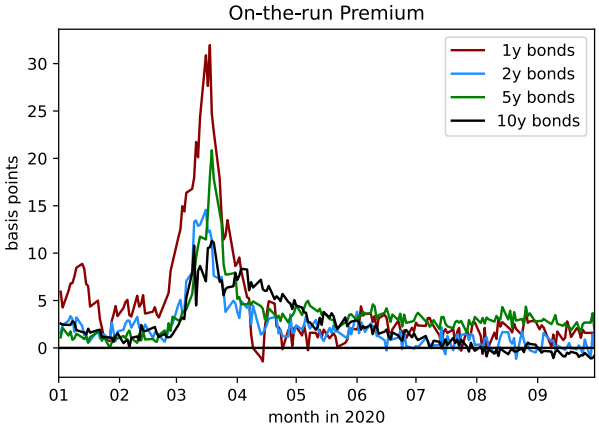



Figure 1: On-the-run premia during the "March 2020" breakdown

deposit or reverse repo facility) generates a certain return on liquid funds between trades for those who have access. This feeds back into the overall cost of trading. If sellers gain access, an increase in the facility rate stimulates trading and prices rise. The stimulated trading implies that more Treasuries end up in the hands of buyers and less in the inventories of primary dealers. Settlement of off-the-run Treasuries is more likely to fail. The reason is that as more Treasuries are sold early, fewer are available during the off-the-run period.

Interestingly, as I found in the data, the premium decreases as the facility rate rises. The reason is intuitive: as off-the-run Treasury prices initially rise, so does the value of the collateral. This leaves buyers better off in the event of default, and buyers increase their demand for the contract with the off-the-run Treasuries. Therefore, the increase in the price of the off-the-run Treasuries is greater than the increase in the price of the on-the-run Treasuries. Also because of this additional demand effect, the increase in the quantity of off-the-run Treasuries traded is initially larger than the increase in the quantity of on-the-run Treasuries.[8] It is worth noting that although I focus on facility access, the result can be interpreted more broadly. Any policy intervention or market change that affects settlement risk by reducing the cost of failing impacts the market and especially the premium in this way.

Paradoxically, in equilibrium, only the primary dealers benefit from a rise in the facility rate, and those who are granted access do not. This is because the primary dealers can now sell more Treasuries at a higher price. By contrast, perfect competition in the contract market erodes any advantage that sellers may have. Finally, buyers of Treasuries lose, first through higher prices and second through an externality. The buyer does not take into account that if he buys more Treasuries early, fewer will be available during their off-the-run period, implying a higher default rate.[9]

**Related Literature** First and foremost, my paper is related to the literature on the on-the-run premium. A well-known framework for the on-the-run premium is provided by Vayanos and Weill (2008).

[8] This policy effect is particularly relevant in the context of the recent crisis, where the market for off-the-run Treasuries froze (Eren and Wooldridge (2021)).

[9] Few of the results depend on search frictions being above a small minimum. See appendix A.2.5.

They have a setup where there are two assets with identical cash flows and agents can go long or short an asset. Since short sellers have to deliver the asset they have borrowed, they face search externalities and favour the asset that is more liquid. Liquidity is self-fulfilling in their model. As in Vayanos and Weill (2008) I also include OTC market frictions and delivery constraints in my model. However, my model is dynamic whereas theirs is static. In addition, I include a key factor to explain the on-the-run premium: the fact that one asset, the off-the-run asset, has been available in the market for a longer time. This also allows for equilibrium selection with a premium on the on-the-run asset, which is not the case in Vayanos and Weill (2008), who have two self-fulfilling equilibria with the premium on either asset.

Another theory of the on-the-run premium comes from Pasquariello and Vega (2009). In their model, the premium arises from endowment shocks. There are two frictions: information heterogeneity and imperfect competition among traders. My model is similar to theirs in the sense that I include uncertainty due to limited information. In my model, the uncertainty is about the stock of off-the-run Treasuries. In their model, it is with respect to on-the-run Treasuries, since the endowment shocks received by agents are private information.

Broadly speaking, compared to Vayanos and Weill (2008) and Pasquariello and Vega (2009), my model focuses on primary dealer inventory and intermediary settlement risk to explain the premium. All dynamics are driven solely by the fact that Treasuries are in the market for a different length of time since issuance. This element implies that the premium is always on the on-the-run asset. The model allows for a more general discussion of the US Treasury market and, in addition, I can also analyse the impact of central bank facility access.

Empirical work attempting to explain the on-the-run premium includes, for example, Strebulaev (2002), Goldreich et al. (2005) and D'Amico and Pancost (2022). Strebulaev (2002) suggests that the premium may measure differences in tax treatment rather than liquidity premia. Goldreich et al. (2005) distinguish between current and future liquidity and suggest that expected future liquidity, not just current liquidity, determines prices and is a significant driver of the on-the-run premium. D'Amico and Pancost (2022) link the on-the-run premium to the risk of unexpected fluctuations in the collateral value of Treasuries.

A recent paper related to mine is Corradin and Maddaloni (2020). They build on Vayanos and Weill (2008) and study central bank intervention. Compared to my paper, they do not study access to facilities but central bank purchases. Specifically, they analyse how purchases by the European Central Bank affected repo specialness in the Italian government bond repo market during the euro area sovereign debt crisis. Specialness is the premium paid to procure a particular security in the repo market. On-the-run securities often trade as "special". Important early work on this topic was done by Duffie (1996) and Krishnamurthy (2002). Corradin and Maddaloni (2020) show that purchases reduce liquidity and increase specialness in the presence of short selling. They also show that assets in high demand and older assets with lower turnover are more likely to fail. The probability of default increases with the specialness of the asset. In contrast, Liu and Wu (2017) show that the on-the-run premium is low when counterparty risk is high. Compared to my model, the risk in Liu and Wu (2017) and Corradin and

Maddaloni (2020) refers to variations in a general risk measure or only in the specific asset. Corradin and Maddaloni (2020) focus exclusively on crisis periods, Liu and Wu (2017) state that their results are particularly pronounced in such periods.

Second, my paper adds to the literature on settlement failures (see, e.g., Fleming et al. (2014), Fleming and Garbade (2002), Fleming and Garbade (2004), Fleming and Garbade (2005), and Garbade et al. (2010)) as well as on non-banks in the Treasury market and their access to the reverse repo facility (see, e.g. Doerr et al. (2023), Eren and Wooldridge (2021), and Frost et al. (2015)). More generally, my work examines the microstructure of the Treasury market and speaks to the literature discussing how to reform it (see, e.g. Duffie (2020), Duffie et al. (2021), Duffie (2023), Durham and Perli (2022), Fleming and Keane (2021), He et al. (2022), Schrimpf et al. (2020), and Vissing-Jorgensen (2021)). The literature on the OTC market environment and dealer markets is also related (see e.g. Duffie et al. (2005), Huh and Infante (2021), Lagos and Rocheteau (2009), and Li and Schürhoff (2018)).

The structure of the paper is as follows: Section 2 describes the US Treasury market, section 3 the environment and section 4 the value functions. The equilibrium is also defined. Section 5 proves the existence of our main equilibrium of interest. Section 6 discusses how the model explains the stylised facts and why the premium is always on the on-the-run Treasury. Section 7 empirically analyses the relationship between the on-the-premium, the settlement fails, and the primary dealer inventories in the data and tests theoretical predictions. The implications of broad access to central bank facilities are analysed in section 8. Given the theoretical results, section 9 analyses the dependence of the on-the-run premium on the reverse repo facility rate in the data. Section 10 presents the Treasury life cycle. Section 11 concludes.

# 2 Description of the Treasury market

In this section I describe the US Treasury market, its structure and trading dynamics, and provide evidence for the stylised facts highlighted in the introduction.

The Treasury spot market is OTC (Fleming et al. (2018)). This means that there is no all-to-all trading at a central venue and no central pricing. Depending on the security traded and the trading partners involved, the degree of friction in the OTC market varies. For example, dealer-to-dealer trading of benchmark on-the-run Treasuries on electronic platforms such as BrokerTec is less frictional than interdealer and dealer-to-customer trading of the less liquid off-the-run Treasuries intermediated on voice and more manually assisted electronic platforms (Bessembinder et al. (2020) and U.S. Department of the Treasury et al. (2015)).[10]

On-the-run Treasuries are the most recently issued Treasuries of a given maturity, and all previously issued Treasuries of the same maturity are referred to as off-the-run. As figure 2a shows for 10-year Treasuries, on-the-run Treasuries trade at significantly lower yields than off-the-run Treasuries with very similar cash flows and maturity dates.[11] They also have higher prices and lower repo rates. The term

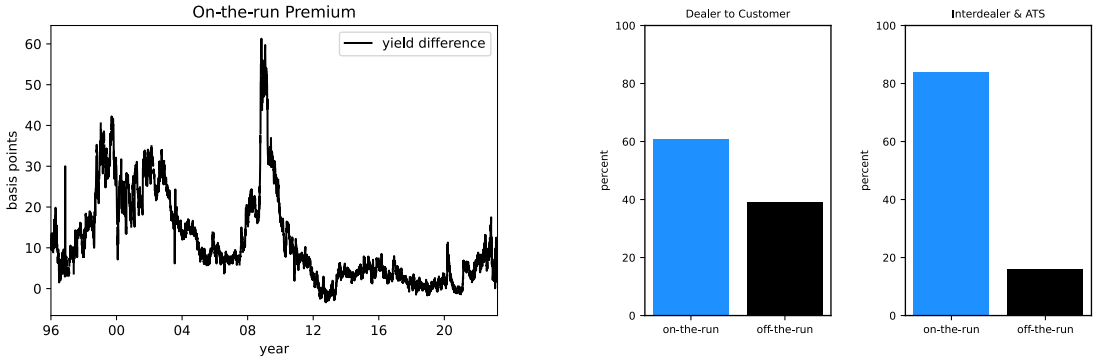
[10] The overall share of trading on all types of electronic platforms in the US Treasury market is 70 percent (Bech et al. (2016)).

[11] The data are taken from the FED yield curve, which is an updated version of the original Gürkaynak-Sack-Wright curve

"very similar" refers to the fact that, in general, there are not two Treasuries in the market with exactly the same cash flow and maturity date where one is on-the-run and the other is off-the-run. In fact, if you want to compare Treasuries with the same overall maturity, it is impossible to do so. In practice, therefore, one either compares Treasuries with the same overall maturity using an estimated off-the-run yield curve (see, for example, the first figure in Christensen et al. (2017)), or one abstracts from small differences in cash flows and maturity dates, or one compares on- and off-the-run Treasuries with the same maturity date that have a different overall maturity (see, for example, Christensen et al. (2020)). In figure 2a, as in Christensen et al. (2017), the on-the-run yield is subtracted from the par yield of seasoned bonds.

Also, despite being more expensive and far fewer in number, on-the-run Treasuries trade in much larger volumes than off-the-run Treasuries, as shown in figure 2b.[12] This is true whether I look at dealer-to-customer or interdealer and automated trading system (ATS) trades.

Another surprising fact is that, on average, USD 40 billion of Treasuries are not delivered on time to settle a contract each day.[13] These events are commonly referred to as "settlement fails". Figure 3a shows the failure rate, which is calculated by dividing the value of Treasuries that failed to be delivered on time by the value of all Treasuries traded. Interestingly, the failure rates differ depending on whether a Treasury is on- or off-the-run, and figure 3b shows that fails involving on-the-run Treasuries are less frequent than those involving off-the-run Treasuries.[14]




(a) On-the-run premium on 10 year Treasury bonds

(b) Trading volumes

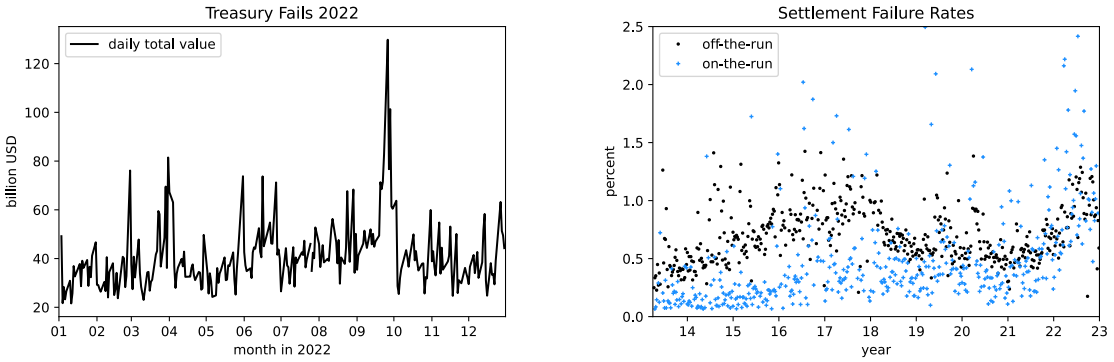
Figure 2: Yields and volumes

(Gürkaynak et al. (2010)), https://www.federalreserve.gov/data/tips-yield-curve-and-inflation-compensation.htm, and the FRB-H15 tables, https://www.federalreserve.gov/releases/h15/.

[12]Trading volumes in on-the-run and off-the-run Treasuries are the volumes reported to TRACE between February and October 2023. The TRACE data is available here:
https://www.finra.org/finra-data/browse-catalog/about-treasury/monthly-file.

[13]The data is provided by the Depository Trust and Clearing Corporation (DTCC) and can be downloaded here: https://www.dtcc.com/charts/daily-total-us-treasury-trade-fails. In times of stress, the daily value can spike. Current policy discussions consider central clearing as an effective way to significantly reduce fails in the future (Fleming and Keane (2021)). For more information on settlement fails, see Fleming and Garbade (2005).

[14]The data are from the FED's primary dealer statistics. It includes outright and financing fails. The median failure rate for on-the-run Treasuries is 0.36% and the median failure rate for off-the-run Treasuries is 0.66%. The rates are not an exact measure. This is because one part of the time series used in the calculation is an average over the reporting week and the other part of the time series reports a value as of the reporting weekday. Given the high frequency, this should not matter and the observed pattern is clear. Each series is outlier adjusted, where an outlier is defined as being below the 2.5% percentile and above the 97.5% percentile. Rates up to 2.5% are shown in the figure. Few rates are higher.




(a) Daily settlement fails

(b) Fails of on- and off-the-run Treasuries

Figure 3: Fails

The OTC structure implies that there are search costs that can explain settlement fails. In particular, search costs become relevant when financial contracts include delivery constraints. For example, spot market trades are often complemented by special repo trades to short-sell specific Treasuries. "Special" refers to the fact that the collateral of the repo is fixed and determined by its number, called ISIN or CUSIP, and the repo may have a rate that differs from the general collateral rate. To short-sell a particular Treasury, it is borrowed using a special repo and sold in the market today. The next day, a Treasury with the same ISIN or CUSIP is bought in the spot market, preferably at a lower price than it was sold on the previous day, and returned to the lender in the repo transaction. If such a Treasury cannot be found, a settlement fail occurs. The borrower of the Treasury in the repo transaction pays a penalty, the Treasury Market Practice Group (TMPG) fail charge.[15] Fleming and Keane (2021) write that on-the-run Treasury fails account for less than a quarter of all fails in non-crisis periods.[16] The figure in Fleming et al. (2014), presented in the appendix A.3, shows that gross fails are much higher in seasoned Treasuries than in others (including on-the-run Treasuries).

Note that arbitrage to exploit the price difference between on- and off-the-run Treasuries involves short selling, but is mostly prohibited by efficient markets because repo rates for them also differ (Krishnamurthy (2002)).

# 3 The environment

Time is discrete and goes on forever, $t = 0, 1, ..., \infty$. The discount factor is $\beta \in (0, 1)$ and each period consists of two subperiods. There are three types of infinitely lived agents in the model: a buyer, a seller, and a primary dealer.[17] There is a continuum in each type.

There are two segmented sequential markets. The first market is called the spot market. It takes place in the first subperiod and is an OTC market. The second market is Walrasian and takes place in

[15] For more information on the TMPG fail charge, see https://www.newyorkfed.org/tmpg and Garbade et al. (2010).

[16] In addition, they also note that on-the-run Treasuries are more often involved in so-called daisy chain fails. One fail implies another as the trades are linked in a chain.

[17] A seller is any financial entity other than a primary dealer. It can for example be a non-bank. A buyer can be interpreted as a long-term holder (e.g. a pension fund).

the second subperiod. It is called contract market. Figure 4 gives an overview over the timeline.

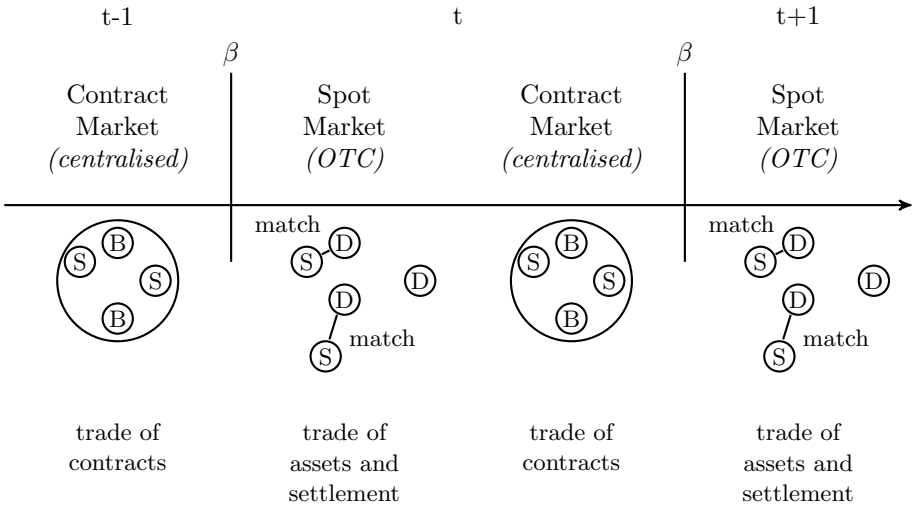



Figure 4: Timeline

There are two goods: a settlement good and real coupons. The settlement good $m \in \mathbb{R}_0^+$ is storable and divisible. Coupons $\delta$ are perishable and are given by two assets: one asset gives one coupon per each second subperiod for two consecutive periods, the other for one period. Assets are storable and divisible. An asset is on-the-run if it belongs to the most recently issued generation of its maturity. Therefore, in each period there are two types of on-the-run and one type of off-the-run assets available for trading: the two-period assets maturing in two periods (2), the two-period assets maturing in one period ($f$), and the one-period assets maturing in one period ($n$). The letters $n$ and $f$ refer to their respective on-the-run ($n$) and off-the-run ($f$) state. I will refer to them henceforth as the on-the-run asset and the off-the-run asset. The two-period asset maturing in two periods is also on-the-run, but its state is not relevant to the analysis.[18] Figure 5 gives an overview over the assets and their cash flow.

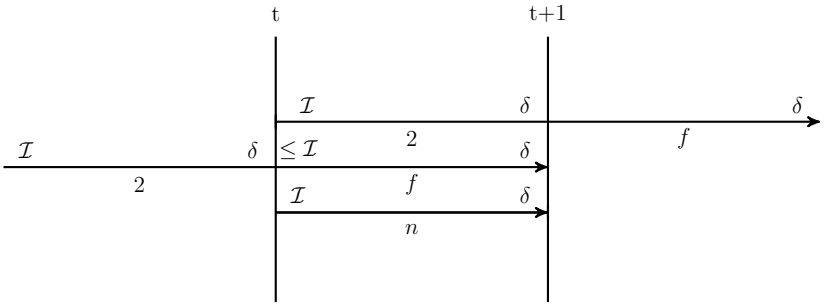



Figure 5: Assets in period $t$

The figure illustrates that the on-the-run ($n$) and off-the-run ($f$) assets are identical in terms of maturity date and coupon. The only difference is the issuance date. Therefore, I compare these two

[18] Only in very few contexts both types of on-the-run assets are meant when using the term on-the-run.

assets to measure the on-the-run premium.[19] At the beginning of each period, primary dealers are endowed with a stock $I^2 = \mathcal{I} \in \mathbb{R}^+$ of newly issued two-period assets and a stock $I^n = \mathcal{I} \in \mathbb{R}^+$ of newly issued one-period assets. The primary dealers' stock $I^f$ of off-the-run assets is endogenous. I assume that buyers and sellers have knowledge of the primary dealers' inventory stock of newly issued assets. They also know the distribution of primary dealers inventories of off-the-run assets (as they know the matching probability described below), but not the inventories of each primary dealer.

All agents have linear utility $\delta$ from consuming the coupons in the second subperiod. Buyers additionally receive utility $g$ each period when holding an asset.[20] Sellers do not value the coupons.[21] All agents have linear utility (disutility) from consuming (producing) the settlement good. The seller can produce it only in the second subperiod. The others can always produce it.[22] It is used to settle trades. It has properties similar to money except that it is a real asset.[23]

In the OTC spot market, primary dealers and sellers trade assets. Primary dealers have a match with probability $(1 - \sigma) > 0$ with a seller. Only matched primary dealers can trade. Sellers always have a match with a primary dealer.[24] Buyers have no access to this market. Primary dealers sell quantities of assets $A^i$ to sellers at price $p^i$, where $i = \{2, f, n\}$. They face an adjustment cost of $\kappa(A^i)$ when selling assets $i$ in terms of settlement good.[25] The function introduces a non-linearity into the model and leads to an interior solution and a determined price.[26] I assume that $\kappa(0) = 0$, $\kappa'(A^i) > 0$, $\kappa''(A^i) > 0$, and that the function is continuous.[27] Possible interpretations of the function are a nonlinear portfolio adjustment cost (see Gârleanu and Pedersen (2013) and Bacchetta and van Wincoop (2021) for examples) or a regulatory cost (see, for example, Macchiavelli and Pettit (2021)).

In every other subperiod so-called contracts are traded by the seller and the buyer. The market is called contract market. The seller sells the contracts to the buyer. A contract is a list $l^i = [a^i, \omega^i, q^i]$. $a^i$ specifies the amount of assets $i$ promised to be delivered in the next subperiod. Sellers cannot commit. $\omega^i a^i$ is the collateral (in settlement good) that the seller has to post at the moment of selling the contract. The buyer has first claim over the collateral in the event of non-delivery.[28] $q^i$ is the contract price in the second subperiod. $q^i a^i$ is the payment (in settlement good) due from the buyer at settlement in the next subperiod. Figure 4 gives an overview over the markets.

[19] The two assets have the same cash flow to maturity and the same maturity date as in Vayanos and Weill (2008) and Pasquariello and Vega (2009). See also, for example, Christensen et al. (2020) for how the premium can be measured. Note that if I would use the two-period on-the-run asset to calculate the premium, I would have to abstract from the second coupon rate as cash flow differences should not be the reason for the premium. The premium and all other results would remain the same.

[20] $g$ can be interpreted as a hedging benefit from holding the asset or simply as a different valuation.

[21] It would not change any results if sellers would value the coupons as much as primary dealers.

[22] This aspect of the model ensures that there is no incentive for the seller to build up settlement goods only to deposit them in the facility. An agreement between a seller and a buyer or a primary dealer whereby the buyer or primary dealer would produce settlement goods for the seller so that the seller could deposit it and pay it back later with a profit is not possible because the seller cannot commit.

[23] The only difference from a nominal model is that the settlement good does not lose or gain value over time due to inflation. The dynamics would not change with a nominal model, and therefore discussing inflation would not add anything relevant.

[24] I make this assumption for simplicity. Changing it would not change the dynamics.

[25] In equilibrium they only sell assets.

[26] The main results also hold without this function. It gets relevant when I discuss access.

[27] Another possibility would be that the function depends on the sum of all assets sold. But I can show that for a positive premium this cannot be the case. Also, I need that the function is increasing and convex.

[28] The collateral in the form of settlement good is similar to the Treasury Market Practice Group (TMPG) fails charge. This fee allows a buyer of Treasuries to claim monetary compensation from the seller if the seller fails to deliver the Treasuries on time. For more information, see https://www.newyorkfed.org/tmpg and Garbade et al. (2010).

In the basic model the seller has access to a central bank facility. The facility can be accessed every first subperiod for one subperiod. Sellers can deposit settlement good and receive an interest rate $r_t$ on it. I assume that $\beta(1+r_t)<1$. This implies that it is not worthwhile to accumulate settlement good one period in advance in order to deposit it in the facility.[29]

# 4 Value functions and equilibrium

## 4.1 Primary dealer

The primary dealer value function at the beginning of the contract market, when holding asset inventories $I_t^2$, $I_t^f$, and $I_t^n$ is

$$V^D(I_t^2=\mathcal{I},I_t^f,I_t^n=\mathcal{I})=\beta(1-\sigma)\Big\{\sum_i\Big[p_t^iA_t^i-\kappa(A_t^i)+\delta(I_t^i-A_t^i)\Big]+\beta V^D(\mathcal{I},\mathcal{I}-A_t^2,\mathcal{I})\Big\}$$
$$+\beta\sigma\Big\{\sum_i\delta I_t^i+\beta V^D(\mathcal{I},\mathcal{I},\mathcal{I})\Big\}.$$

With a probability of $(1-\sigma)$, the primary dealer has a match with a seller in the spot market and can sell assets. Optimal prices and quantities are determined by the bargaining problem described below. For each kind of asset $i\in\{2,f,n\}$, the primary dealer sells the optimal amount $A_t^i$. When selling the amount $A_t^i$ he receives the price $p_t^i$ for each of them and faces the cost $\kappa(A_t^i)$. In addition, he cannot consume any future coupons of these assets but only of his inventory left, $(I_t^i-A_t^i)$. With probability $\sigma$ the primary dealer has no match and consumes the coupons of his inventory of assets. The inventory of off-the-run assets, $I_t^f$, depends on whether the primary dealer had a match in the previous period and, if so, how much was traded. The inventory is therefore endogenous. The primary dealer is also endowed with the newly issued assets $I_t^2$ and $I_t^n$. As they are just issued, their inventory is of size $\mathcal{I}$. The three inventory quantities are the state variables.

The amounts $A_t^i$ sold and the price $p_t^i$ are determined by a bargaining problem between the primary dealer and the seller. I assume that the primary dealer has full bargaining power and the seller does not make any profits.[30] This means that the primary dealer sets the price just as high such that the seller is indifferent between delivery and non-delivery, i.e. $\left(q_{t-1}^i-p_t^i+\omega_{t-1}^i\right)a_t^i=q_{t-1}^ia_t^i$. The price therefore equals the collateral value $\omega_{t-1}^i$:

$$p_t^i=\omega_{t-1}^i\qquad\forall\, i \text{ and } t. \tag{1}$$

The primary dealer maximises his trade surplus. The Lagrange function to his maximisation problem in each period $t$ is given by:[31]

[29] I could in addition assume that primary dealers also have access to the facility, but this does not change the dynamics. This is why I am omitting it to ease notation.

[30] Where relevant, I relax this assumption later.

[31] If the primary dealer does not sell the two-period assets today, then the assets remain in his inventory in the next period if he has no match, or if he has a match, I assume that a part is sold and the rest remains in his inventory as well. This means that his inventory constraint on the off-the-run assets is not binding if he still has the full inventory available in the next period. I will show later why I assume that this holds in equilibrium and that I can always find equilibria where it does. The surplus can be written as $p_t^iA_t^i-\kappa(A_t^i)-\delta A_t^i-\beta\left[\sigma\delta A_t^2+(1-\sigma)\left(p_{t+1}^fA_{t+1}^f-\kappa(A_{t+1}^f)+\delta\left(A_t^2-A_{t+1}^f\right)\right)\right]$.

$$\mathcal{L}(\{A_t^i, p_t^i, \lambda_t^i, \widetilde{\lambda}_t^i\}_i) = \sum_i \left(p_t^i A_t^i - \kappa(A_t^i) - \delta A_t^i\right) - \beta\delta A_t^2 + \lambda_t^i \left[I_t^i - A_t^i\right] + \widetilde{\lambda}_t^i \left[a_t^i - A_t^i\right] + \beta(1-\sigma)\lambda_{t+1}^f \left[\mathcal{I} - A_t^2\right]. \tag{2}$$

The trade surplus is given by the income generated, $p_t^i A_t^i$, minus the costs $\kappa(A_t^i)$ and the opportunity costs in terms of coupons, $-\delta A_t^i - \beta\delta A_t^2$. Note that for the asset maturing in two periods, the primary dealer takes into account that if he sells the asset today, he not only forgoes the coupon today but also tomorrow. For each type of asset, the primary dealer faces an inventory constraint, $A_t^i \leq I_t^i$. He cannot sell more than what he has. In addition, he cannot sell more than what the seller is willing to buy of each type of asset given by $a_t^i$. Therefore the following constraint needs to hold: $A_t^i \leq a_t^i$. The primary dealer makes a take-it-or-leave-it offer to the seller subject to his delivery constraints. He sets the price as such that the seller is just as well off with the purchase as without it. In both cases the seller receives the contract price $q_{t-1}^i$. If he delivers he has to buy the asset at price $p_t^i$ but he can keep his collateral $\omega_{t-1}^i$.

The first order conditions are:

$$\begin{aligned} p_t^2 &= \kappa'(A_t^2) + \delta + \beta\delta + \lambda_t^2 + \widetilde{\lambda}_t^2 + \beta(1-\sigma)\lambda_{t+1}^f \\ p_t^f &= \kappa'(A_t^f) + \delta + \lambda_t^f + \widetilde{\lambda}_t^f \\ p_t^n &= \kappa'(A_t^n) + \delta + \lambda_t^n + \widetilde{\lambda}_t^n. \end{aligned} \tag{3}$$

The prices equal the marginal costs faced when selling the assets and take into account potentially binding constraints. $\lambda_t^i$ is the Lagrange multiplier of the inventory constraint $A_t^i \leq I_t^i$. $\widetilde{\lambda}_t^i$ is the Lagrange multiplier of the demand constraint $A_t^i \leq a_t^i$.

The complementary slackness conditions are:

$$\begin{aligned} \lambda_t^i(I_t^i - A_t^i) &= 0 \qquad \forall\, i \text{ and } t \\ \widetilde{\lambda}_t^i(a_t^i - A_t^i) &= 0 \qquad \forall\, i \text{ and } t. \end{aligned}$$

The inventories of the newly issued assets equal $\mathcal{I}$, i.e. $I_t^2 = I_t^n = \mathcal{I}$. The inventory of off-the-run assets $I_t^f$ can take two values. The primary dealers who did not had a match the period beforehand have an inventory of $I_t^{f,h} \equiv I_t^f = \mathcal{I}$. The one who did have a match have an inventory of $I_t^{f,l} \equiv I_t^f = \mathcal{I} - A_{t-1}^2$.[32] The letter $h$ stands for high and the letter $l$ for low, corresponding to the higher and lower inventories, respectively. Given the probability of having no match is $\sigma$, by the law of large numbers a share $\sigma$ of primary dealers has an inventory of $I_t^{f,h}$ and a share $(1-\sigma)$ has an inventory of $I_t^{f,l}$. I denote the Lagrange multiplier of the inventory constraint of the high inventory group $\lambda_t^{f,h}$ and that of the low inventory group $\lambda_t^{f,l}$. I denote the sold off-the-run assets of the high inventory group $A_t^{f,h}$ and those of

[32] For the sake of simplicity, trading between primary dealers is not considered in this model. We could also model trade between primary dealers. As long as there are frictions in the interdealer market that lead to primary dealers not fully balancing their inventories, the results are valid. See for example Eisfeldt et al. (2023) for an analysis of price dispersion between dealers and intermediation capacity.

the other group $A_t^{f,l}$. The other Lagrange multipliers are the same for both groups. In the following I assume that $\lambda_t^n = \lambda_t^2 = \lambda_t^{f,h} = 0$. The equilibrium where this holds is our main equilibrium of interest. In section A.1 I discuss other equilibria. In these other equilibria, the main dynamics are the same. They are extreme cases of the main equilibrium.

## 4.2 Buyer

The buyer and seller anticipate which of the primary dealers' inventory constraints are non-binding and which are potentially binding when trading. They do this because they know the distribution over inventories. As mentioned above, I assume that $\lambda_t^n = \lambda_t^2 = \lambda_t^{f,h} = 0$. This means that primary dealers are unconstrained in selling assets if they still have the full amount of the assets in their inventory. Given this assumption, I have to distinguish two cases: $\lambda_t^{f,l} > 0$ and $\lambda_t^{f,l} = 0$. A fraction $(1-\sigma)$ of primary dealers has already sold a part of its stock of assets in the previous period and if $\lambda_t^{f,l} > 0$ they face a demand for assets today that will exhaust the remaining stock. If $\lambda_t^{f,l} = 0$ the demand is lower than the remaining stock.

The buyer's value function at the beginning of the contract market is

$$\begin{aligned} V^b(a_{t-1}^2) = \max_{\{a_t^i\}_i} \sum_i & -\beta q_{t-1}^i a_t^i + \beta(\delta+g)\left(\sum_i a_t^i + a_{t-1}^2\right) \\ & + \beta(1-\sigma)\left[\omega_{t-1}^f - (\delta+g)\right](a_t^f - I_t^{f,l})\mathbb{I}_{\lambda_{t+1}^{f,l}>0} + \beta V^b(a_t^2). \end{aligned}$$

The buyer's state variable is $a_{t-1}^2$. These are the assets he bought last period and which did not mature yet. For each asset, the buyer chooses how many he wants to buy from the seller. For each asset he wants to buy, he must build up the payment $q_{t-1}^i a_t^i$ in the form of settlement goods the next period at settlement. The assets are delivered in the next period at settlement, and the buyer receives utility $(\delta+g)$ from each asset he holds. If I am in the case where $\lambda_t^{f,l} > 0$, then with a probability $(1-\sigma)$ his seller encounters a primary dealer who is constrained in his inventory of off-the-run assets and only $I_t^{f,l}$ instead of $a_t^f$ assets are delivered. For the amount of assets for which there is a settlement failure $(a_t^f - I_t^{f,l})$, he receives the collateral $\omega_{t-1}^f a_t^f$.

The first order conditions are

$$\begin{aligned} q_{t-1}^2 &\geq (1+\beta)(\delta+g) \\ q_{t-1}^n &\geq (\delta+g) \\ q_{t-1}^f &\geq (\delta+g) + (1-\sigma)\left[\omega_{t-1}^f - (\delta+g)\right]\mathbb{I}_{\lambda_{t+1}^{f,l}>0}. \end{aligned} \tag{4}$$

The prices are greater or equal the discounted marginal utilities. The price of the two-period on-the-run asset is twice the price of the one-period on-the-run asset before adjusting for discounting. If I am in the case where settlement fails can occur, i.e. $\lambda_t^{f,l} > 0$, then the price of the off-the-run asset also reflects the risk of a settlement failure.

## 4.3 Seller

The seller's value function at the beginning of the contract market is[33]

$$V^s = \max_{\{a_t^i\}_i} \sum_i -\omega_{t-1}^i a_t^i + \beta(1+r_t)\sum_i (q_{t-1}^i - p_t^i + \omega_{t-1}^i)a_t^i$$
$$+ \beta(1+r_t)(1-\sigma)(p_t^f - \omega_{t-1}^f)(a_t^f - I_t^{f,l})\mathbb{I}_{\lambda_{t+1}^{f,l}>0} + \beta V^s.$$

The seller chooses the optimal number of contracts to sell to the buyer. That is, he chooses for each asset the amount he is willing to deliver in the next period. He has to build up collateral $\omega_{t-1}^i a_t^i$ in the form of settlement goods to support a contract. $\omega_t^i$ is taken as given. In the next period, the seller goes to the spot market and buys the assets. For each asset he can deliver, he receives the price $q_{t-1}^i$ from the buyer, he pays the spot market price $p_t^i$ to the primary dealer, and he can keep his accumulated collateral (all in the form of settlement goods).[34] Any remaining funds after the trade, he can deposit in the central bank facility and receive an interest rate $r$ on them.[35] If I am in the case where $\lambda_t^{f,l} > 0$, then with probability $(1-\sigma)$ he is matched with a primary dealer who is constrained and can only deliver $I_t^{f,l}$ instead of $a_t^f$. For this amount of non-deliverable assets $(a_t^f - I_t^{f,l})$, the seller's collateral is seized and given to the buyer. The seller does not buy this amount on the spot market and therefore does not have to pay the spot market price.

The first order conditions are

$$\begin{aligned}
&\omega_{t-1}^2 \geq \beta(1+r_t)(q_{t-1}^2 - p_t^2 + \omega_{t-1}^2) \\
&\omega_{t-1}^f \geq \beta(1+r_t)(q_{t-1}^f - p_t^f + \omega_{t-1}^f) + \beta(1+r_t)(1-\sigma)(p_t^f - \omega_{t-1}^f)\mathbb{I}_{\lambda_{t+1}^{f,l}>0} \\
&\omega_{t-1}^n \geq \beta(1+r_t)(q_{t-1}^n - p_t^n + \omega_{t-1}^n).
\end{aligned} \tag{5}$$

The collateral value that has to be built up today is greater or equal to the contract price he receives tomorrow and the value of the collateral he can keep minus the spot price he has to pay to acquire the asset. If $\lambda_t^{f,l} > 0$ non-delivery occurs with probability $(1-\sigma)$ and in this case he does not have to pay the spot price but he cannot keep the collateral.

## 4.4 Equilibrium and premium definition

Next, I define the equilibrium.

**Definition 1** (**Equilibrium**). *An equilibrium consists of*

*a) the contract and spot prices of all assets ($q_{t-1}^i \ \forall i$ and $p_t^i \ \forall i$),*

*b) the assets contracted ($a_t^i \ \forall i$) and sold ($A_t^2, A_t^n, A_t^{f,h}$, and $A_t^{f,l}$),*

[33] The seller never buys assets for himself. The reason is that there are negative gains from trade because the primary dealer and the seller value the asset the same but if they were to trade, they would face the adjustment cost.

[34] The seller delivers an asset if $p_t^i \leq \omega_{t-1}^i$, which is the case in equilibrium for all $i$ and $t$. This means that the value of the collateral seized in case of non-delivery must be as high as the value of the assets he buys on the spot market. Since this constraint is always satisfied (see section 4.1), it is not added to the maximisation problem.

[35] In equilibrium, the funds are not negative.

*c) the collateral values ($w^i_{t-1}$ $\forall i$)*

*and the primary dealer, the buyer, and the seller behave optimally given contract prices $q^i_{t-1}$ and collateral values $w^i_{t-1}$ ((3), (4), (5)) and the delivery constraints (1) are satisfied.*

Given the equilibrium definition I make two additional assumptions. First, I assume that if the buyer and the seller are indifferent to buying more or less assets (after accounting for the probability of a settlement failure), I assume that they are willing to contract the maximum amount of assets that is profitable for the primary dealers to sell (if they are not constrained). This implies that in each equilibrium

$$\begin{aligned} a^2_t &= A^2_t \\ a^n_t &= A^n_t \\ a^f_t &= A^{f,h}_t. \end{aligned} \tag{6}$$

Note that in any equilibrium, as soon as an inventory constraint starts to bind, so does the corresponding constraint on the contracts. Also, if one is slack, the other is slack. The only exception is $\widetilde{\lambda}^{f,l}_t$, which can be zero even if $\lambda^{f,l}_t > 0$, but not vice versa. Therefore, as I concentrate on equilibria where $\lambda^n_t = \lambda^2_t = \lambda^{f,h}_t = 0$, then also $\widetilde{\lambda}^n_t = \widetilde{\lambda}^2_t = \widetilde{\lambda}^{f,h}_t = 0$.

Second, I assume in the following that due to perfect competition and market regulation the contract price and the collateral values adjust in equilibrium such that the first order conditions of the buyer and the seller hold with equality. This means that there is a non-zero finite amount of contracts sold in all assets, $a^i_t \in (0, \infty)$ $\forall i$.

Next, I define the on-the-run premium.

**Definition 2** (**On-the-run premium**). *The on-the-run premium is defined as $\Delta_t \equiv p^n_t - p^f_t$.*

As mentioned above, the on-the-run and off-the-run assets have the same cash flow to maturity and mature on the same day. The only difference is their issuance date. To measure the on-the-run premium I compare these two assets. A positive (negative) premium implies that the yield to maturity of the on-the-run asset is lower (higher) than that of the off-the-run asset.

# 5 Existence

This section focuses on the case where $\lambda^2_t = \lambda^n_t = \lambda^{f,h}_t = 0$. This means that primary dealers are unconstrained if they still have the full stock of assets available, i.e. $\mathcal{I} > max(A^2_t, A^n_t, A^{f,h}_t)$. As argued in the previous section 4.4 $\widetilde{\lambda}^n_t = \widetilde{\lambda}^2_t = \widetilde{\lambda}^{f,h}_t = 0$ holds as well.

I argue that in an equilibrium where there is a non-zero premium, it must be the case that $\lambda^{f,l}_t > 0$ and $\widetilde{\lambda}^{f,l}_t = 0$. Therefore in this equilibrium $\mathcal{I} \in (max(A^2_t, A^n_t, A^{f,h}_t),\ A^2_{t-1} + A^{f,h}_t)$.[36] This means that primary dealers who sold some of their inventory in the previous period and can sell again today, are constrained. All other primary dealers are not constrained.

[36] Note that I could also add the knife-edge case where $\mathcal{I} = max(A^2, A^n, A^{f,h})$ and $\lambda^2_t = \lambda^n_t = \lambda^{f,h}_t = 0$. To make the notation easier, I omit it.

**Proposition 1.** *In an equilibrium where $\mathcal{I} > max(A_t^2, A_t^n, A_t^{f,h})$, a necessary condition for the on-the-run premium to be non-zero is $\mathcal{I} < A_{t-1}^2 + A_t^{f,h}$.*

*Proof.* See appendix A.2.1. □

I need constrained primary dealers for an equilibrium with a positive premium, because this gives rise to settlement fails. The fails imply the premium (see section 6). Without settlement fails, both assets are priced the same, since they are perfect substitutes in this case.[37] Therefore, our equilibrium candidate is the equilibrium where $\lambda_t^{f,l} > 0$.

**Proposition 2.** *A necessary and sufficient condition for the existence of the equilibrium with $\mathcal{I} \in (max(A_t^2, A_t^n, A_t^{f,h}), A_{t-1}^2 + A_t^{f,h})$ is $\sigma > \frac{1-\beta(1+r_t)}{\beta(1+r_t)} \frac{\delta}{g}$, which implies positive trade in all assets. I can always find issuance sizes $\mathcal{I}$ where this equilibrium exists.*

*Proof.* See appendix A.2.2. □

For positive demand in off-the-run assets, the probability of a settlement fail, $(1-\sigma)$, cannot be too high. Therefore the condition. Given the condition of the proposition is satisfied, I show in the proof that I can always find an $\mathcal{I}$ where our equilibrium of interest exists, i.e. $\mathcal{I} \in (max(A_t^2, A_t^n, A_t^{f,h}), A_{t-1}^2 + A_t^{f,h})$. I call this equilibrium from now on "premium equilibrium".

**Definition 3** (**Premium equilibrium**). *The premium equilibrium is the equilibrium where $\lambda_t^{f,l} > 0$ and all other Lagrange multipliers are zero.*

I will restrict the further analysis to the premium equilibrium. In the appendix A.1, I discuss other equilibria. The dynamics and intuition are the same as in the premium equilibrium.

I summarise the main result of this section as follows: For an equilibrium with a premium, I need that some primary dealers are inventory constrained. The equilibrium always exists if the probability to find the off-the-run assets is high enough such that buyers and sellers want to trade it.

# 6 Premium equilibrium

In this section I analyse and discuss the premium equilibrium. To motivate my theory, I first show the scatter plot between the 10 year on-the-run premium and the net outright positions of primary dealers in 10 year Treasury bonds in figure 6.[38] I observe that there is a negative correlation between the on-the-run premium and the net outright positions of the primary dealers with tight 95% confidence bands. In my theory primary dealer inventory risk will be key to explain the on-the-run premium.

[37] I use a linear utility function, but this is true for any utility function where the assets are perfect substitutes, i.e., only the sum of the two assets matters.

[38] The data sources for the on-the-run premium are the same as in figures 2a. The net positions of primary dealers can be downloaded from the FED's Primary Dealer Statistics, https://www.newyorkfed.org/markets/counterparties/primary-dealers-statistics. The frequency is weekly. The data are deflated using the "Consumer Price Index for All Urban Consumers: All Items in U.S. City Average", which can be downloaded from FRED. I set the index to 1 when the series starts in 2010.

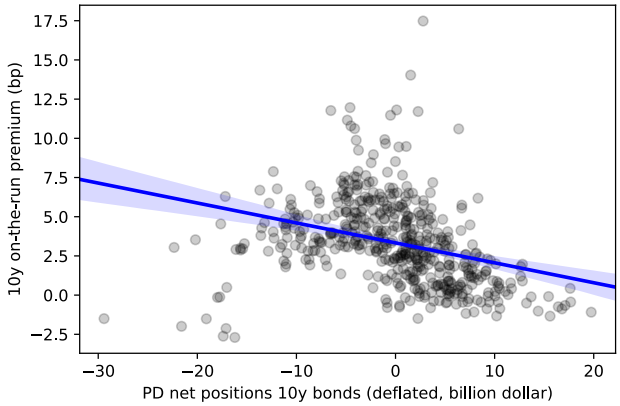



Figure 6: The correlation between the premium and the net positions

## 6.1 Graphical example with the two-period asset

To discuss the relevant dynamics in the premium equilibrium, I illustrate the life cycle of two two-period assets issued in period $t$ in the figure 7 below (dark blue dots).

In period $t-1$ the buyer buys a contract from the seller. In period $t$ the assets are issued and on-the-run. Every primary dealer receives one asset in his inventory. I restrict here the inventory to one asset for illustrative purposes. On the spot market the seller is matched with one of the two primary dealers, buys the asset and delivers it to the buyer against a payment in the form of settlement good (not illustrated).[39] The other primary dealer was not matched with a seller and keeps his asset in his inventory. After the spot market, the contract market takes place in the second subperiod. The buyer buys again one contract promising the delivery of this asset. In period $t+1$ the assets are off-the-run (as new assets are issued).[40] On the OTC spot market, the seller is matched with the primary dealer who was able to sell his asset already the period beforehand. A settlement fail occurs. Nevertheless it is optimal for the buyer to initially buy one contract. He takes the probability of a settlement fail into account when taking his decision.

[39] The asset stays in the portfolio of the buyer until it matures in $t+1$.

[40] In my model the assets are always off-the-run after one period because new assets are issued (here depicted by the light blue dots). For simplicity the buyer does not buy any contract promising the delivery of these new assets in this example.

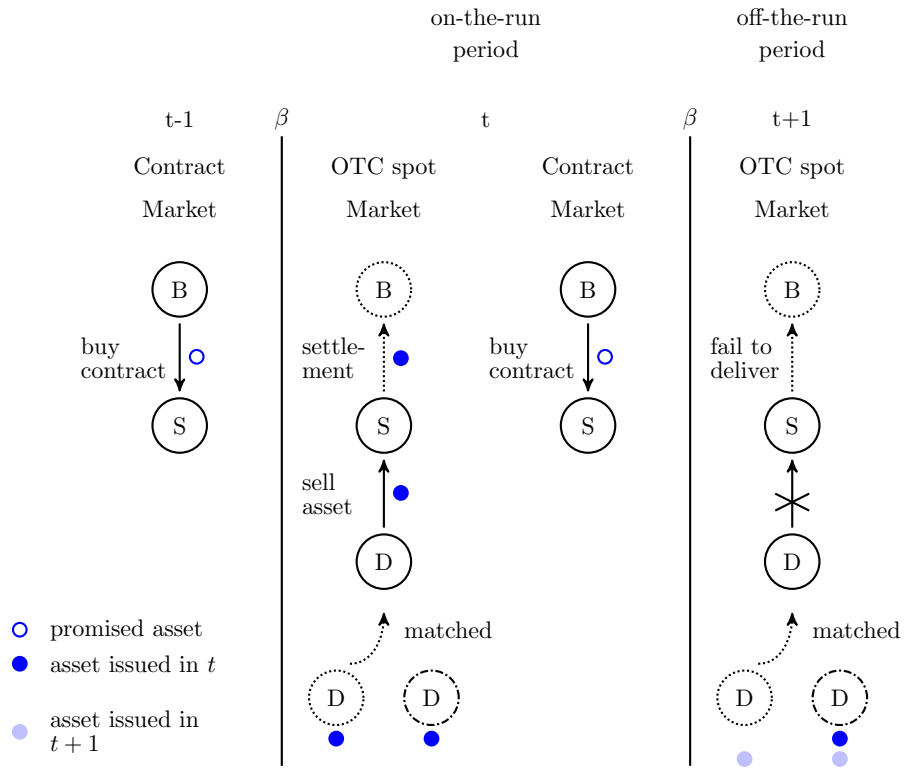



Figure 7: Dynamics

The figure shows only one kind of asset, the two-period asset. In the model there is also the one-period on-the-run asset. It's easy to see that settlement fails occur for off-the-run assets, but not for the on-the-run assets. Inventories do not differ for on-the-run assets because they are less long in the market. As I show in the next subsection, in equilibrium there is a premium for on-the-run assets because they do not fail to settle compared to off-the-run assets where the probability of failure is priced in.

To sum up, the time since issuance is the only feature that distinguishes the two assets. And it is precisely this difference, combined with OTC market frictions and delivery constraints, that leads to settlement fails and hence the premium.[41] Finally, it is worth pointing out that off-the-run assets are scarcer (see figure). However, it is uncertainty, not scarcity per se, that causes the premium. A scarce asset that could be bought without uncertainty would not lead to fails and the premium.

[41] Without settlement fails, both assets would be priced only according to the marginal utility that the coupons (incl. additional utility $g$) give to the buyer. This is true not only for any linear utility function like the one used here, but also for any non-additively separable non-linear function. Once a buyer has obtained the assets, there is no reason why the on- and off-the-run assets should not be substitutes, given that they have the same coupons and are held to maturity.

## 6.2 Equilibrium prices, quantities, premium, and fails

In the premium equilibrium, the following equations for prices and quantities hold:

$$\begin{aligned} p_t^n &= \beta(1+r_t)(\delta+g) \\ p_t^f &= \frac{\sigma}{1-(1-\sigma)\beta(1+r_t)}\beta(1+r_t)(\delta+g) \\ p_t^2 &= (1+\beta)\beta(1+r_t)(\delta+g) \end{aligned}$$

and

$$\begin{aligned} A_t^n &= \kappa'^{-1}\left[\beta(1+r_t)(\delta+g)-\delta\right] \\ A_t^{f,h} &= \kappa'^{-1}\left[\frac{\sigma}{1-(1-\sigma)(1+r_t)\beta}\beta(1+r_t)(\delta+g)-\delta\right] \\ A_t^{f,l} &= \mathcal{I} - A_{t-1}^2 \\ \kappa'(A_t^2) &= W + \beta(1-\sigma)\kappa'(\mathcal{I}-A_t^2) \end{aligned}$$

where $W \equiv \left[(1+\beta)-\beta(1-\sigma)\frac{\sigma}{1-(1-\sigma)\beta(1+r_t)}\right]\beta(1+r_t)(\delta+g)-(1+\beta\sigma)\delta$.

I can observe from the equations that the spot prices directly depend on the buyer's "valuation" of the assets $(\delta+g)$. The on-the-run price of the two-period asset is $(1+\beta)$ times the on-the-run price of the one-period asset, because the buyer receives twice utility from it. All of my results with respect to the premium would also hold if I would compare the two-period off-the-run asset to the two-period on-the-run asset and abstract from the cash flow in the second period to make them equal in terms of cash flow.

The quantity of the two-period asset maturing in two periods and the off-the-run asset traded by constrained dealers depends on the issue size $\mathcal{I}$. The reason for the first mentioned fact is that when these two-period assets are sold, it is taken into account that less can be sold tomorrow due to the binding inventory constraint. This binding inventory constraint is then also the reason why the amount of off-the-run assets traded by constrained primary dealers depends on $\mathcal{I}$.

Lastly I derive the on-the-run premium. I deduct $p_t^f$ from $p_t^n$. This gives rise to the premium according to the following proposition:

**Proposition 3.** *The on-the-run premium is given by:* $\Delta = \left[1-\frac{\sigma}{1-(1-\sigma)\beta(1+r_t)}\right]\beta(1+r_t)(\delta+g) > 0$. *If* $\sigma \to 1$, *then* $\Delta \to 0$.

The on-the-run premium depends on the buyer's asset "valuation" $(\delta+g)$, the probability to find the off-the-run assets $\sigma$, and $\beta(1+r_t)$ which is the discount factor cost of the seller for binding collateral. If assets would be found with certainty in the second period, i.e. $\sigma \to 1$, then the premium vanishes.[42]

It is important to point out that key for a positive on-the-run premium to arise is

$$I_t^{f,h} \neq I_t^{f,l}.$$

[42]The premium also vanishes if $\sigma = 0$ but then I am not anymore in the premium equilibrium and the formula above does not hold.

In the premium equilibrium, unconstrained primary dealers have a full inventory, i.e. $I_t^{f,h} = \mathcal{I}$, and constrained ones have a reduced inventory, i.e. $I_t^{f,l} = \mathcal{I} - A_{t-1}^2$ with $A_{t-1}^2 > 0$. Therefore, $I_t^{f,h} > I_t^{f,l}$. It is this difference in inventories that leads to the uncertainty that implies settlement fails. The fails themselves imply the premium. Compared to Vayanos and Weill (2008) the premium is always on the on-the-run asset.

Lastly, I define the settlement failure rate of asset $i$ as the value of assets $i$ involved in a fail divided by the overall amount of assets promised to be delivered:

$$f_t^i \equiv \frac{p_t^i \mathbb{I}_{a_t^i > I_t^i} \mathbb{P}_t^i (a_t^i - I_t^i)}{p_t^i a_t^i}$$

where $\mathbb{P}_t^i$ is the failure probability.[43] As $\lambda_t^{f,h} = 0$ it follows that $f_t^n = 0$ and as $\lambda_t^{f,l} > 0$ it follows that $f_t^f = \frac{(1-\sigma)p_t^f(a_t^f - I_t^f)}{p_t^f a_t^f} = (1-\sigma)\left(1 - \frac{\mathcal{I} - a_t^2}{a_t^f}\right)$.[44]

Our equilibrium is consistent with the four stylised facts described in the introduction. First, on-the-run assets are more expensive than off-the-run assets (positive on-the-run premium). Second, they trade in larger volumes. Third, settlement fails occur and lastly, off-the-run assets fail to settle more often. I show this in the following proposition:

**Proposition 4.** *In the premium equilibrium*

$$\begin{aligned} p_t^n &> p_t^f \\ A_t^n &> \sigma A_t^{f,h} + (1-\sigma) A_t^{f,l} \\ f_t^f &> 0 \\ f_t^f &> f_t^n. \end{aligned}$$

*Proof.* See appendix A.2.3. □

In equilibrium not only $p_t^n > p_t^f$ but also $A_t^n > \sigma A_t^{f,h} + (1-\sigma) A_t^{f,l}$, i.e. on-the-run assets not only have a higher price but are also traded in larger quantities. Both equilibrium results can be explained by the fact that off-the-run assets are less attractive because they fail to settle more often. It is contrary to common intuition that a scarcer asset has the lower price, but this observation is consistent with what is observed in the market (U.S. Department of the Treasury (2022)).

I summarise the above discussion as follows: The on-the-run premium is due to differences in inventories of off-the-run assets as they are longer in the market. The reason is as follows: Some of the off-the-run assets are locked up in buy-and-hold portfolios because they have already been sold during their on-the-run period. Since not all primary dealers faced the same demand during the on-the-run period due to the OTC market structure, there are differences in their inventories at the start of the off-the-run period. This implies uncertainty about the amount of assets available in an upcoming match

[43] An alternative way to analyse settlement fails would be to compare the overall value of assets involved in fails (see e.g. figure 13 in the appendix A.3).

[44] In the data I also observe some fails with on-the-run assets. It would be straightforward to also generate them in the model by having a non-uniform distribution across dealers with buyers being aware of the distribution but not the individual holdings. As the frictions would still imply that the dispersion increases over time, the result (more fails with off-the-run assets compared to on-the-run assets) would still hold.

with them in the OTC market. This leads to a higher frequency of settlement fails for off-the-run assets, as contracts promising their delivery cannot always be fulfilled. There is a preference for on-the-run assets because their settlement is not risky. Compared to off-the-run assets they are safe in this aspect. This implies that they carry a premium, i.e. are more expensive on the spot market, and trade in larger quantities than off-the-run assets.

# 7 On-the-run premium, settlement fails, and inventories in the data

Next, I analyse whether my theory is consistent with the empirical evidence. From the theory I derive two hypotheses, which I test:

**Hypothesis 1.** *A higher failure rate leads to a higher premium.*

**Hypothesis 2.** *Lower primary dealer inventories lead to a higher failure rate.*

The first hypothesis follows directly from proposition 3. There I show that if the probability of finding the assets, $\sigma$, goes to 1 (no fails occur), then the premium vanishes. Otherwise, there are settlement fails that lead to the premium. Therefore, in a first step, I regress on-the-run premium data on the failure rate of off-the-run assets. My theory predicts a negative coefficient.

Second, I conjecture that lower primary dealer inventories lead to settlement fails. To test the second hypothesis, I regress the failure rate of off-the-run assets on the net outright positions in Treasuries of the primary dealers. Lower net outright positions are expected to capture inventory uncertainty. Given my theoretic results, I should observe a negative coefficient.

I first examine the relationship between the failure rate and the on-the-run premium. For the outcome variable in the first regression (see table 1), I use data of the 1, 2, 3, 5, 7, 10, 20, and 30 year on-the-run premia. Each maturity, denoted by $m$ and day, denoted by $t$, in the sample is a separate observation. The explanatory variable, the failure rate of off-the-run Treasuries, is not available for different maturities. The explanatory variable is therefore the same for each on-the-run premium maturity. I run three regressions. In the first, I only regress on the failure rate. In the second, I add other control variables. The additional control variables are the logarithm of the VIX, the 10 year - 2 year yield spread, and the general collateral financing repo rate. In the third regression, I add additionally crisis and maturity fixed effects. The crisis fixed effects capture the months October 2014, September 2019, and March 2020. In these months there was a crisis in the Treasury market (see U.S. Department of the Treasury et al. (2015), Anbil et al. (2020), and Schrimpf et al. (2020)). As we use weekly data, the crisis dummies are equal to one in each week within these months. The maturity fixed effects capture each maturity $m$ of

the on-the-run premia. The regression equation (3) is given by

$$\begin{aligned}\text{On-the-run premium}_{t,m} = \alpha_m &+ \beta_1 \text{ Failure rate off-the-run Treasuries}_t + \beta_2\, ln(\text{VIX}_t) \\ &+ \beta_3 \text{ 10 year - 2 year yield spread}_t \\ &+ \beta_4 \text{ General collateral financing repo rate}_t \\ &+ \beta_6\, \mathbb{I}_{10/14,t} + \beta_7\, \mathbb{I}_{09/19,t} + \beta_8\, \mathbb{I}_{03/20,t} + u_t.\end{aligned}$$

The results are shown in table 1. As expected, the coefficient of the failure rate is positive. It is significant at the 5% level (with and without additional control variables and fixed effects). If I do not control for any other variables, then an increase in the failure rate by 1 percentage point, increases the premium by 2.2 basis points, which is about half a standard deviation. The size of the effect is almost the same if I add the additional control variables.

| | (1) Premium | (2) Premium | (3) Premium |
|---|---|---|---|
| Failure rate off-the-run Treasuries | 2.2483*** | 2.4206*** | 2.3150*** |
| | (0.630) | (0.563) | (0.503) |
| lnVIX | | 1.1706*** | 1.1653 *** |
| | | (0.570) | (0.492) |
| 10 year - 2 year yield spread | | 0.2605 | 0.1965 |
| | | (0.364) | (0.346) |
| General collateral financing repo rate | | 0.0128 | -0.0364 |
| | | (0.314) | (0.287) |
| Constant | 1.1459*** | -2.5491 | -2.0929 |
| | (0.412) | (2.054) | (1.878) |
| Crisis and maturity fixed effects | No | No | Yes |
| No. Observations: | 3760 | 3760 | 3760 |
| R-squared: | 0.017 | 0.025 | 0.219 |
| Adj. R-squared: | 0.017 | 0.024 | 0.216 |

Notes: I use Newey-West standard errors with 6 lags. *** indicates significance at the 5% level. ** indicates significance at the 10% level. * indicates significance at the 15% level. The sample period is April 2013-2022 and the frequency is weekly.
The data source on the premia (in basis points) is the same as in figure 2a. The failure rate data (in percent) is the same as in figure 3a. The VIX data are taken from FRED. The 10 year and 2 year yields (par yields of seasoned bonds in percent) are taken from the FED yield curve, which is an updated version of the original Gürkaynak-Sack-Wright curve (Gürkaynak et al. (2010)), https://www.federalreserve.gov/data/tips-yield-curve-and-inflation-compensation.htm. The general collateral financing repo rate (in percent) is provided by the Depository Trust and Clearing Corporation (DTCC) and can be downloaded here: https://www.dtcc.com/charts/dtcc-gcf-repo-index. The crisis fixed effects capture the months October 2014, September 2019, and March 2020 and the maturity fixed effects each maturity of the on-the-run premia (1y, 2y, 3y, 5y, 7y, 10y, 20y, and 30y).

Table 1: Premium on off-the-run failure rate regression

Next, I examine the relationship between the failure rate and dealer inventories. I regress the failure rate of off-the-run Treasuries on the net outright positions of primary dealers in Treasuries of different maturity baskets (see table 2). The maturity of the baskets is denoted by $k$. Specifically, I have the following baskets for the net outright positions in Treasuries: below or equal 3 years, above 3 years to 6 years, above 6 years to 11 years, above 11 years. Each basket and day is a separate observation. I

run the regression first without and then with additional control variables. The control variables are the same as in the previous regression (see table 1). In a third additional regression, I add crisis fixed effects also the same ways as before. The regression equation (3) is given by

$$\begin{aligned}\text{Off-the-run failure rate}_t = \alpha &+ \beta_1 \text{ Primary dealer net positions}_{t,k} + \beta_2\, ln(\text{VIX}_t) \\ &+ \beta_3 \text{ 10 year - 2 year yield spread}_t \\ &+ \beta_4 \text{ General collateral financing repo rate}_t \\ &+ \beta_6\, \mathbb{I}_{10/14,t} + \beta_7\, \mathbb{I}_{09/19,t} + \beta_8\, \mathbb{I}_{03/20,t} + u_t.\end{aligned}$$

| | (1) Off-the-run failure rate | (2) Off-the-run failure rate | (3) Off-the-run failure rate |
|---|---|---|---|
| PD net positions (deflated baskets) | -0.0012*** | -0.0031*** | -0.0030*** |
| | (0.001) | (0.000) | (0.000) |
| lnVIX | | -0.0769*** | -0.0786*** |
| | | (0.033) | (0.033) |
| 10 year - 2 year yield spread | | -0.1559*** | -0.1637*** |
| | | (0.019) | (0.019) |
| General collateral financing repo rate | | -0.0256** | -0.0324*** |
| | | (0.014) | (0.014) |
| Constant | 0.7208*** | 1.1508*** | 1.1673*** |
| | (0.015) | (0.111) | (0.110) |
| Crisis fixed effects | No | No | Yes |
| No. Observations: | 1912 | 1880 | 1880 |
| R-squared: | 0.009 | 0.132 | 0.145 |
| Adj. R-squared: | 0.008 | 0.130 | 0.142 |

I use Newey-West standard errors with 6 lags. *** indicates significance at the 5% level. ** indicates significance at the 10% level. * indicates significance at the 15% level. The sample period is April 2013-2022 and the frequency is weekly.
The data source for the net outright positions (in billion US dollars) is the same as in figure 6. The data are also deflated in the same way (with the index set to 1 when the series starts in April 2013). The failure rate data (in percent) is the same as in figure 3a. The VIX data are taken from FRED. The 10 year and 2 year yields (par yields of seasoned bonds in percent) are taken from the FED yield curve, which is an updated version of the original Gürkaynak-Sack-Wright curve (Gürkaynak et al. (2010)), https://www.federalreserve.gov/data/tips-yield-curve-and-inflation-compensation.htm. The general collateral financing repo rate (in percent) is provided by the Depository Trust and Clearing Corporation (DTCC) and can be downloaded here: https://www.dtcc.com/charts/dtcc-gcf-repo-index. The crisis fixed effects capture the months October 2014, September 2019, and March 2020.

Table 2: Failure rate on net positions regression

The results are shown in table 2. As expected, the coefficient of the primary dealers net positions is negative and significant at the 5% level.[45] Adding the additional control variables and fixed effects does not change the result. As the net positions are denominated in dollars, the coefficient is best interpreted by first multiplying it by the standard deviation of the net outright positions. If the net positions increase by one standard deviation, then the failure rate reduces by 2, 6, resp. 5 basis points (regression 1-3).[46]

To provide further evidence consistent with my theory, I examine the volatility of the net positions

[45] The result still holds if the first difference in the primary dealers' net positions is taken.
[46] This corresponds to 0.1 (regression 1), 0.2 (regression 2), respectively, standard deviations.

of primary dealers in Treasuries.[47] I find evidence in the data consistent with my theory that off-the-run inventories are much more affected by search and matching frictions than on-the run inventories. Table 3 shows the volatilities. Primary dealer inventories in on-the-run Treasuries are much less volatile than inventories including all kind of Treasuries. This holds across all maturity baskets. This is consistent with my theory if I assume that there is a slight variation in $\sigma$ over time, which must be the case in reality. The effect of the search and matching frictions on off-the-run inventories can then be observed. They are volatile, while on-the-run inventories are much more stable.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| On-the-run | Maturity | 2y | 3y | 5y | 7y | 10y | 30y |
| | Volatility | 4.1 | 4.0 | 3.5 | 2.7 | 4.1 | 2.3 |
| All | Maturity | $\leq$3y | (3y,6y] | (6y,7y] | (7y,11y] | $\geq$11y | |
| | Volatility | 21.2 | 11.7 | 7.6 | 7.0 | 12.3 | |

Table 3: Primary dealer net positions volatilities

# 8 Central bank facility access

How does broadening access to a central bank's reverse repo or deposit facility to any type of intermediary affect the Treasury market (in normal times)? How do prices, premia, traded quantities, fails, and profits change? This section provides answers to these questions.

In my model, the central bank facility is comparable to a deposit facility or a reverse repo facility, where I abstract from the collateral provided. The collateral part would not make the existing dynamics disappear. First, the repos from the facility are general collateral repos. For such repos the cash lender is willing to accept any collateral that falls into a broad class, and is not looking for any particular collateral (Bowman et al. (2017)). Second, even if the facility were to provide a specific Treasury sought, the Treasury would have to be returned the next day and would only temporarily ease availability.

To motivate my theory I show the scatter plot between the 10 year on-the-run premium and the reverse repo facility rate in figure 8.[48] I observe that there is a negative correlation between the 10 year on-the-run premium and the reverse repo facility rate with tight 95% confidence bands. As I show below, this empirical evidence is consistent with my theory.

## 8.1 Homogeneous sellers

First, I analyse the situation where all sellers have or gain access to the central bank facility. The facility provides liquidity in-between transactions. It can be accessed every first subperiod for one subperiod.

[47] The data can be downloaded from the FED's Primary Dealer Statistics, https://www.newyorkfed.org/markets/counterparties/primary-dealers-statistics. The data are deflated using the "Consumer Price Index for All Urban Consumers: All Items in U.S. City Average", which can be downloaded from FRED. I set the index to 1 when the on-the-run net positions time series start in April 2013. I use data from April 2013 to the end of 2022. The frequency is weekly. To calculate the volatilities, I use the average of the data over the full time horizon.

[48] The data source for the premium is the same as in figure 2a. The reverse repo facility rate data is provided by the New York FED and can be downloaded here: https://www.newyorkfed.org/markets/omo_transaction_data#rrp. I take daily averages. The time horizon is 23 September 2013-2021 Q2 and the frequency is weekly.

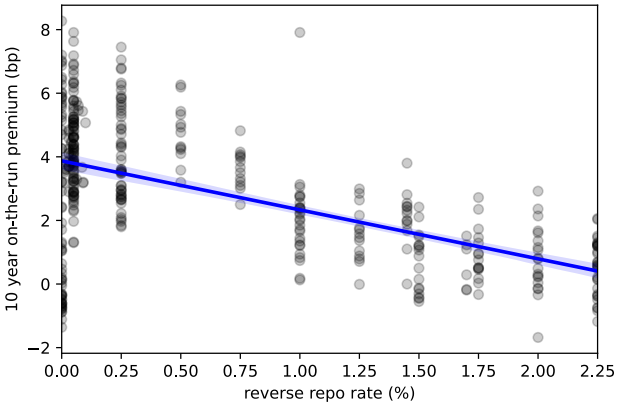



Figure 8: The correlation between the premium and the reverse repo facility rate

Sellers can deposit settlement good and receive an interest rate $r_t$ on it. Sellers do this because they have a positive net settlement good position after the trades: They received the contract price $q_t^i$ and had to pay (in case of delivery) the lower spot price $p_t^i$ and could keep the collateral $\omega_t^i$. The seller represents any type of financial institution (e.g., a hedge fund or a non-primary dealer). The goal of the analysis is to find, in a general setting, the effect of the reverse repo or deposit facility rate on trading (in normal times) when any type of financial firm other than a primary dealer is given access to the facility.

I still look at the premium equilibrium. I assume a permanent unanticipated increase in the facility rate $r_t$ at the beginning of the contract market in $t = \widetilde{t} - 1$. In addition to the situation where all sellers already had access and the facility rate increases, the increase can also represent the situation where all sellers gain access to the facility and because of the facility now face a higher interest rate than before (with no interest or a lower market rate). I assume that there is still a positive premium after the sellers gain access. The results of the analysis can be summarised as follows:

**Corollary 1.** *An unanticipated and permanent increase in $r_t$ in the second subperiod in $t = \widetilde{t} - 1$*

*a) increases all spot prices, $\frac{dp_t^i}{dr_{\widetilde{t}-1}} > 0 \; \forall i$ and $\forall t \geq \widetilde{t}$,*

*b) decreases the on-the-run premium, $\frac{d\Delta_t}{dr_{\widetilde{t}-1}} < 0 \; \forall t \geq \widetilde{t}$,*

*c) increases the quantities of on-the-run and off-the-run assets traded if no inventory constraint binds, $\frac{dA_t^n}{dr_{\widetilde{t}-1}} > 0$ and $\frac{dA_t^{f,h}}{dr_{\widetilde{t}-1}} > 0 \; \forall t \geq \widetilde{t}$,*

*d) implies that the quantities of off-the-run assets traded initially, i.e. in $t = \widetilde{t}$, stay the same and then decrease if inventory constraints bind, $\frac{dA_t^{f,l}}{dr_{\widetilde{t}-1}} = 0$ for $t = \widetilde{t}$ and $\frac{dA_t^{f,l}}{dr_{\widetilde{t}-1}} < 0$ $\forall t > \widetilde{t}$ if $\sigma > \widetilde{\sigma}$,*

*e) implies that overall more assets are offloaded from the inventories and end up in the portfolio of the buyer, i.e. the holder with the highest marginal asset valuation, $\frac{d(A_t^2 + A_t^n + \sigma A_t^{f,h} + (1-\sigma) A_t^{f,l})}{dr_{\widetilde{t}-1}} > 0$ for $t = \widetilde{t}$ and $\forall t > \widetilde{t}$ if $\sigma > \widetilde{\sigma}$,*

*f) increases the settlement failure rate of the off-the-run asset,* $\frac{df_t^f}{dr_{\widetilde{t}-1}} > 0$ *for* $t = \widetilde{t}$ *and* $\forall t > \widetilde{t}$ *if* $\sigma > \widetilde{\sigma}$.

*Proof.* See appendix A.2.5 □

The value $\widetilde{\sigma}$ is the value of $\sigma$ above which always (but not only) $\frac{\partial A_t^2}{\partial r_{\widetilde{t}-1}} > 0 \ \forall t \geq \widetilde{t}$. I define it in appendix A.2.5 and show that with a reasonable calibration it is an empirically very small value.

The intuition for this result is as follows: An increase in the facility rate increases the profitability of the trade for the seller. The facility is always available, and excess liquidity can be deposited until the seller enters the next trade. The facility is not a substitute for trading, but a complement to it. The higher the interest rate, the higher the profitability. Increased profitability leads to a positive supply shock in the contract market. This in turn implies a positive demand shock in the spot market. In equilibrium, spot prices and quantities traded by unconstrained primary dealers increase.

Off-the-run prices and quantities react more strongly than their on-the-run equivalents, and the premium falls. The reason is that the case of non-delivery for the off-the-run asset is less costly than before because the collateral values increase due to the rise in prices. This reduces the spread between the real valuation of holding the asset and the collateral. This effect additionally triggers the demand for off-the-run assets. This makes the policy particularly interesting in the context of the Treasury market crisis during the pandemic, where the market for off-the-run assets froze (Eren and Wooldridge (2021)).

The amount of off-the-run assets traded by constrained dealers remains the same in the first period after the rate hike, because inventories are determined from the previous period. Later, it decreases because the inventories of the constrained dealers are smaller because more assets have already been sold during their on-the-run period (if search frictions are above a small minimum value of $\widetilde{\sigma}$). This is also the main reason for the observed increase in the settlement failure rate of off-the-run assets. The assets are not available later and more fails occur. Nevertheless, due to the interest rate increase, more assets end up in the portfolio of the buyer, the agent with the highest marginal asset valuation.

Putting all the above results in a broader context, I conclude that any kind of policy that lowers the costs of trade and intermediation can trigger the effects described. Access is one possibility.

The next result is about who benefits from an increase in the facility rate $r_t$. I look at the impact on the lifetime values of the buyer, the seller, and the primary dealer. There are two groups of primary dealers: One group of primary dealers has an inventory of $\mathcal{I}$ off-the-run assets and another group has an inventory of $\mathcal{I} - A_t^2$. I take the lifetime value of both groups and average them according to their proportions in the population. To simplify the notation, I define $V_{\widetilde{t}-1}^{D,a} \equiv (1-\sigma)V_{\widetilde{t}-1}^{D}(\mathcal{I} - A_t^2) + \sigma V_{\widetilde{t}-1}^{D}(\mathcal{I})$.

**Corollary 2.** *An unanticipated and permanent increase in* $r_t$ *in the second subperiod in* $t = \widetilde{t} - 1$

*a) does not affect the lifetime value of sellers,* $\frac{dV_{\widetilde{t}-1}^s}{dr_{\widetilde{t}-1}} = 0$,

*b) decreases the lifetime value of buyers,* $\frac{dV_{\widetilde{t}-1}^b}{dr_{\widetilde{t}-1}} < 0$,

*c) increases the lifetime value of primary dealers,* $\frac{dV_{\widetilde{t}-1}^D}{dr_{\widetilde{t}-1}} > 0$ *if* $\sigma > \widetilde{\sigma}$.

*Proof.* See appendix A.2.6. □

As pointed out above, see appendix A.2.5 for the definition of $\widetilde{\sigma}$.

The intuition for the result is as follows: Competition among sellers in the contract market erodes any positive profits for them to zero. Initially increased profitability is offset by higher spot prices in equilibrium.

Primary dealers benefit from the policy by providing the assets that are in higher demand. They sell more assets, they sell them earlier, and they sell them at a higher price. In equilibrium, primary dealers sell assets until their marginal nonlinear cost equals the price. Therefore, each asset sold yields a small positive marginal surplus until the last asset is sold, where the marginal surplus equals zero. Since prices are higher in the new equilibrium, breakeven is reached at a higher quantity of assets. Primary dealers profits increase.

Buyers' utility falls. To gain intuition, I first explain how buyers' utility or benefit materialises in equilibrium. Since the price of the off-the-run asset reflects the utility of the last unit bought, it incorporates the probability that a settlement fail occurs. But the first part of the bought assets is found with certainty, the part $I_t^f$. The value the buyer places on these assets is therefore higher. Nevertheless, in equilibrium he pays the same price for all the assets and therefore he has a small benefit.

An increase in the facility rate increases the off-the-run price and decreases the quantity of off-the-run assets found with certainty (as the quantity of on-the-run assets traded with a two-period maturity increases). Both effects lead to a lower profit. In summary, buyers' profits decrease as the spread between the value of an uncertain unit and a certain unit decreases and there are fewer certain units. The decrease in profits or utility is an externality problem. The buyer does not consider how the purchase of two-period assets affects the availability and price of the same assets in the next period. This is in contrast to the primary dealer, who manages his inventory and takes into account that a two-period asset sold today cannot be sold in the next period.

I solved my baseline model under the assumption that the primary dealer has full bargaining power. I can relax this assumption and prove that even with positive bargaining power of the seller, in equilibrium, the seller does not make a profit in contrast to the primary dealer.

**Corollary 3.** *Result 2 still holds even with positive bargaining power of sellers.*

When maximising the joint surplus of both agents, the first-order conditions of the spot market problem change to:

$$
\begin{aligned}
p_t^2 &= \kappa'(A_t^2) + \delta + \beta\delta + (\omega_{t-1}^2 - p_t^2) + \lambda_t^2 + \widetilde{\lambda}_t^2 + \beta(1-\sigma)\lambda_{t+1}^f \\
p_t^f &= \kappa'(A_t^f) + \delta + (\omega_{t-1}^f - p_t^f) + \lambda_t^f + \widetilde{\lambda}_t^f \\
p_t^n &= \kappa'(A_t^n) + \delta + (\omega_{t-1}^n - p_t^n) + \lambda_t^n + \widetilde{\lambda}_t^n.
\end{aligned}
$$

The surplus is divided according to their bargaining power: $\left(\omega_{t-1}^i - p_t^i\right) A_t^i = \frac{(1-\theta)}{\theta} S_t^i$ where $S_t^i$ is the surplus of the primary dealer when selling assets $i$. The first-order conditions of the seller and the buyer when making their decision about the optimal number of contracts to sell and buy do not change. This is crucial because I see from the seller's first-order conditions and his value function that he makes no profit in equilibrium. Even if in equilibrium $\omega_{t-1}^2 - p_t^2 > 0$ (due to the new pricing scheme) and the seller

receives part of the positive total surplus of the OTC trade, the collateral value $\omega_{t-1}^i$ in the Walrasian contract market adjusts in such a way that he has no total profits. Otherwise, the seller would supply an infinite number of contracts or no contracts at all. I assumed that this is not the case in equilibrium, which is a reasonable assumption. Therefore, even with positive bargaining power, the seller never profits from an increase in the facility rate if he has access.

## 8.2 Heterogeneous sellers

In this subsection I assume that there is a measure $\xi$ of sellers which have access to the facility and a measure $(1-\xi)$ which does not have access. Sellers which have access are denoted by $a$ and the ones which don't by $na$. The facility rate is given by $r_t$ and the market rate by $r_t^m$. I assume that $r_t > r_t^m$. Agents with no access face the market rate, while the others will use the facility. Again I will look at the impact of sellers gaining access. But beforehand I present the equilibrium equations.

I again consider the premium equilibrium. I first show the prices and quantities of the newly issued assets. Analogously to section 6, the contract and spot prices are given by

$$\begin{aligned} q_t^n &= (\delta + g) \\ p_t^{n,na} &= \beta(1 + r_t^m)(\delta + g) \\ p_t^{n,a} &= \beta(1 + r_t)(\delta + g) \end{aligned}$$

and

$$\begin{aligned} q_t^2 &= (1+\beta)(\delta + g) \\ p_t^{2,na} &= (1+\beta)\beta(1 + r_t^m)(\delta + g) \\ p_t^{2,a} &= (1+\beta)\beta(1 + r_t)(\delta + g). \end{aligned}$$

The traded quantities are determined by the following equations:

$$\begin{aligned} A_t^{n,na} &= \kappa'^{-1}\left[\beta(1 + r_t^m)(\delta + g) - \delta\right] \\ A_t^{n,a} &= \kappa'^{-1}\left[\beta(1 + r_t)(\delta + g) - \delta\right] \end{aligned}$$

and

$$\begin{aligned} \kappa'(A_t^{2,na}) &= W^{na} + \beta(1-\sigma)\kappa'(\mathcal{I} - A_t^{2,na}) \\ \kappa'(A_t^{2,a}) &= W^{a} + \beta(1-\sigma)\kappa'(\mathcal{I} - A_t^{2,a}) \end{aligned}$$

where
$W^{na} \equiv p_t^{2,na} - \beta(1-\sigma)\left[\xi p_t^{f,a} + (1-\xi)p_t^{f,na}\right] - (1+\beta\sigma)\delta$ and
$W^{a} \equiv p_t^{2,a} - \beta(1-\sigma)\left[\xi p_t^{f,a} + (1-\xi)p_t^{f,na}\right] - (1+\beta\sigma)\delta.$

Next I show the off-the-run prices and quantities. Let me denote by $\mathbb{P}^a$ the probability of a seller with access finding the off-the-run assets. Analogously, I define $\mathbb{P}^{na}$ as the probability for the sellers

without access. Analogously to section 6, the off-the-run prices are given by

$$p_t^{f,na} = \frac{\mathbb{P}^{na}}{1-(1-\mathbb{P}^{na})\beta(1+r_t^m)}\beta(1+r_t^m)(\delta+g).$$
$$p_t^{f,a} = \frac{\mathbb{P}^{a}}{1-(1-\mathbb{P}^{a})\beta(1+r_t)}\beta(1+r_t)(\delta+g)$$

As in section 5 I assume that the primary dealers with a full inventory are unconstrained. Therefore

$$A_t^{f,na,h} = \kappa'^{-1}\left[\frac{\mathbb{P}^{na}}{1-(1-\mathbb{P}^{na})\beta(1+r_t^m)}\beta(1+r_t^m)(\delta+g)-\delta\right].$$
$$A_t^{f,a,h} = \kappa'^{-1}\left[\frac{\mathbb{P}^{a}}{1-(1-\mathbb{P}^{a})\beta(1+r_t)}\beta(1+r_t)(\delta+g)-\delta\right]$$

Lastly, I define the on-the-run premium by $\Delta \equiv [\xi p_t^{n,a} + (1-\xi)p_t^{n,na}] - [\xi p_t^{f,a} + (1-\xi)p_t^{f,na}]$. For a positive premium either $\mathbb{P}^a$ or $\mathbb{P}^{na}$ or both must be below 1. If they are below one, they either equal $\sigma$ or $\sigma + (1-\sigma)(1-\xi)$ given that $(\mathcal{I} - A_{t-1}^{2,na}) > (\mathcal{I} - A_{t-1}^{2,a})$, or $A_{t-1}^{2,a} > A_{t-1}^{2,na}$.[49]

Next, I analyse the effect of sellers without access gaining access. The analysis is done analogously to section 8.1. As in section 8.1 I also assume that there is still a positive premium after all sellers have access. The results are summarised below.

**Corollary 4.** *If sellers without access gain access, meaning that their interest rate unanticipated permanently increases from $r_t^m$ to $r_t$ in the second subperiod in $t = \widetilde{t} - 1$, then*

*a) all on-the-run spot prices charged by the sellers gaining access increase and all sellers charge $p_t^{j,a}$ in $t \geq \widetilde{t}$ for $j = 2$ and $n$,*

*b) off-the-run spot prices charged by the sellers gaining access initially, i.e. in $t = \widetilde{t}$, increase and all sellers charge $p_{\widetilde{t}}^{f,a}$,*

*c) the on-the-run premium $\Delta_t$ initially, i.e. in $t = \widetilde{t}$, decreases,*

*d) all quantities of on-the-run assets traded by the sellers gaining access increase and all sellers trade $A_t^{j,a}$ in $t \geq \widetilde{t}$ for $j = 2$ and $n$,*

*e) quantities of off-the-run assets traded by the sellers gaining access inititally, i.e. in $t = \widetilde{t}$, increase if no inventory constraints bind and all sellers trade $A_{\widetilde{t}}^{f,a,h}$,*

*f) quantities of off-the-run assets traded by the sellers gaining access initially, i.e. in $t = \widetilde{t}$, stay the same if inventory constraints bind and equal $A_{\widetilde{t}-1}^{f,na,l}$.*

*The points b), c) and e) also hold for $t > \widetilde{t}$ if the probabilities to find the off-the-run assets do not change, otherwise the effects are ambiguous.*

[49] For the inventories I do not only need to distinguish if a primary dealer has met a seller the previous period or not (as before) but also if the match was with a seller with or without access. Depending on the type, more or less assets were sold and therefore the inventory differs. A mass $\sigma$ of agents has a high inventory of $\mathcal{I}$ because they did not face any demand the period beforehand. A mass $(1-\sigma)\xi$ of primary dealers has sold assets to a seller with access in the previous period and they have an inventory of $(\mathcal{I} - A_{t-1}^{2,a})$. Lastly, a mass $(1-\sigma)(1-\xi)$ of sellers have an inventory of $(\mathcal{I} - A_{t-1}^{2,na})$ as they met a seller without access the previous period.

*Proof.* See appendix A.2.7. □

As already pointed out in section 8.1, the value $\widetilde{\sigma}$ is the value of $\sigma$ above which always (but not only) $\frac{\partial A_t^2}{\partial r_{\tilde{t}-1}} > 0 \; \forall t \geq \widetilde{t}$. I define it in appendix A.2.5. I also show that with a reasonable calibration it is an empirically very small value.

The result is self-explanatory. The direction of the effects on the sellers who gain access is (initially at $t = \widetilde{t}$ and also later if the probabilities of finding the off-the-run assets do not change) analogous to our baseline scenario with homogeneous sellers in section 8.1.

# 9 On-the-run premium and facility rate in the data

In this section I examine the relationship between the on-the run premium and the central bank facility rate. Based on corollary 1 I test the following hypothesis:

**Hypothesis 3.** *A higher reverse repo facility rate leads to a lower premium.*

I then regress on-the-run premia data on the reverse repo facility rate (see table 4).[50] My theory predicts a negative coefficient. I use data on the 1, 2, 3, 5, 7, 10, 20, and 30 year on-the-run premia. Each maturity and each day in the sample is a separate observation. The explanatory variable is the reverse repo facility rate. The first observation I use is from the 23 September 2013, as this was the first day the facility was available on a large scale.[51] I run my regressions with and without additional control variables. The additional control variables are the deflated reverse repo facility trade amounts, the logarithm of the VIX, the 10 year - 2 year yield spread, and the general collateral financing repo rate. I also add crisis and maturity fixed effects in a third regression the same way as in regression 1. The regression equation (3) is given by

$$\begin{aligned}\text{On-the-run premium}_{t,m} = \alpha_m &+ \beta_1 \text{ Reverse repo facility rate}_t \\ &+ \beta_2 \text{ Reverse repo facility trade amounts}_t + \beta_3 \, ln(\text{VIX}_t) \\ &+ \beta_4 \text{ 10 year - 2 year yield spread}_t \\ &+ \beta_5 \text{ General collateral financing repo rate}_t \\ &+ \beta_6 \, \mathbb{I}_{10/14,t} + \beta_7 \, \mathbb{I}_{09/19,t} + \beta_8 \, \mathbb{I}_{03/20,t} + u_t.\end{aligned}$$

From table 4, I can see that as expected, the coefficient of the reverse repo facility rate is negative. A one percentage point increase in the reverse repo facility rate reduces the on-the-run premium by 0.01 basis points (regression 1), 2.2 basis points[52] (regression 2 and 3), respectively. The coefficient is only significant (at the 5% level) when we add the additional control variables.[53]

[50] I use the reverse repo facility and not any deposit facility data as the access discussion centers around this one. An increasing number of institutions already access this facility (Frost et al. (2015), Baklanova et al. (2015) and Marte (2021)).

[51] See https://www.newyorkfed.org/markets/opolicy/operating_policy_130920.html for more information.

[52] This corresponds to around 0.7 standard deviations.

[53] The data on each reverse repo facility transaction, including the rate, the amount traded and other details, are only publicly available after two years. However, a reverse repo facility rate series can be downloaded without any time lag. When I use these data (without controlling for trade size) and extend the time horizon to 31 August 2023, the coefficient of the rate is positive in the regression without the additional control variables and still negative in the one with. Both coefficients are insignificant.

| | (1) Premium | (2) Premium | (3) Premium |
|---|---|---|---|
| Reverse repo facility rate | -0.0101 | -2.2225*** | -2.1755*** |
| | (0.070) | (0.446) | (0.372) |
| Reverse repo facility trade amounts | | 0.1498*** | 0.1186*** |
| | | (0.048) | (0.047) |
| lnVIX | | 0.8326*** | 0.7957*** |
| | | (0.321) | (0.287) |
| 10 year - 2 year yield spread | | 0.2183 | 0.0898 |
| | | (0.161) | (0.115) |
| General collateral financing repo rate | | 1.6331*** | 1.4591*** |
| | | (0.406) | (0.358) |
| Constant | 2.7820*** | -0.1528 | 1.1022 |
| | (0.093) | (1.051) | (0.886) |
| Crisis and maturity fixed effects | No | No | Yes |
| No. Observations: | 14952 | 13076 | 13076 |
| R-squared: | 0.000 | 0.051 | 0.061 |
| Adj. R-squared: | 0.000 | 0.051 | 0.060 |

I use Newey-West standard errors with 6 lags. *** indicates significance at the 5% level. ** indicates significance at the 10% level. * indicates significance at the 15% level. The sample period is 23 September 2013-2021 Q2 and the frequency is daily. The data source for the premium (in basis points) is the same as in figure 2a. The reverse repo facility rate and trade amount data are provided by the New York FED and can be downloaded here: https://www.newyorkfed.org/markets/omo_transaction_data#rrp. I take daily averages. The rate is in percent, the trade amount in billion US dollars. I deflate the trade amounts using the "Consumer Price Index for All Urban Consumers: All Items in U.S. City Average", which can be downloaded from FRED. I set the index to 1 when the trade amount series starts on the 23 September 2013. The VIX data are taken from FRED. The 10 year and 2 year yields (par yields of seasoned bonds in percent) are taken from the FED yield curve, which is an updated version of the original Gürkaynak-Sack-Wright curve (Gürkaynak et al. (2010)), https://www.federalreserve.gov/data/tips-yield-curve-and-inflation-compensation.htm. The general collateral financing repo rate (in percent) is provided by the Depository Trust and Clearing Corporation (DTCC) here: https://www.dtcc.com/charts/dtcc-gcf-repo-index. The crisis fixed effects capture the months October 2014, September 2019, and March 2020 and the maturity fixed effects each maturity of the on-the-run premia (1y, 2y, 3y, 5y, 7y, 10y, 20y, and 30y).

Table 4: Premium on reverse repo facility rate regression

# 10 Life cycle model

This section presents an extension of the basic model. It shows that the model can be used as a tool to describe the life cycle of a Treasury. In addition I can explain two more stylised facts:

*I When-issued Treasuries trade at a premium compared to previously issued Treasuries.*

*II The primary market prices are lower than the secondary market prices.*

The observations are illustrated in the appendix A.3. The life cycle of a Treasury can be divided into three periods as illustrated in figure 9: the when-issued period, the on-the-run period, and the off-the-run period. The auction takes place between the announcement and the issuance of the assets. Each period and the auction are characterised by a different price. The chart below illustrates the life cycle.

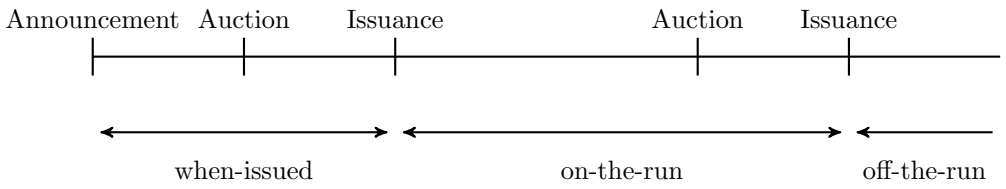



Figure 9: Life cycle

The when-issued market takes place after the auction of the security is announced but before it is issued. The when-issued market is important for price discovery (Durham and Perli (2022)). One possible interpretation of the prices $q_t^2$ and $q_t^n$ is as the when-issued price of the respective assets. Thus, the model covers all three periods with its prices. The fourth important price during the life cycle, the auction price, is added to the model below. Since I know the on- and off-the-run prices, I can derive the auction price. I still look at the premium equilibrium.

I will first discuss the auction price of the one-period asset. A primary dealer taking part in the auction chooses to bid with a bundle consisting of price and quantity which I denote by $\left\{p_t^{1,A}, A_t^{1,A}\right\}$. The bundle specifies the quantity the primary dealer wants to buy and the price he is willing to pay. For each possible price $p_t^{1,A}$ that could be chosen, the optimal quantity is given by the solution to the following maximisation problem:

$$\max_{A_t^{1,A}} -p_t^{1,A}A_t^{1,A} + (1-\sigma)\left[p_t^n A_t^n - \kappa(A_t^n) + \delta(A_t^{1,A} - A_t^n)\right] + \sigma\delta A_t^{1,A}$$

s.t.

$$A_t^{1,A} \geq A_t^n.$$

If the primary dealer buys an asset at the auction, he must pay the price $p_t^{1,A}$ today. With probability $(1-\sigma)$ he can enter the market in the same subperiod and sell the quantity $A_t^n$ of the asset. With probability $\sigma$ he will hold the asset until maturity and consume the coupon. I know that in the premium equilibrium the constraint is not binding. Therefore, $p_t^{1,A} = \delta$. The price must equal the marginal utility of the asset. Otherwise there is infinite or no demand. Bidding will therefore drive the price to this value. The supply side is given by the Treasury, which auctions the amount $I_t^n = \mathcal{I}$. Therefore, in equilibrium, $A_t^{1,A} = I_t^n = \mathcal{I}$ and $p_t^{1,A} = \delta$.

The maximisation problem for the two-period asset is analogous:

$$\begin{aligned}\max_{p_t^{2,A},A_t^{2,A}} & -p_t^{2,A}A_t^{2,A} + (1-\sigma)\Big\{p_t^2 A_t^2 - \kappa(A_t^2) + \delta(A_t^{2,A} - A_t^2) \\ & + \beta(1-\sigma)\left[p_{t+1}^f A_{t+1}^{f,l} - \kappa(A_{t+1}^{f,l}) + \delta(A_t^{2,A} - A_t^2 - A_{t+1}^{f,l})\right] + \beta\sigma\delta(A_t^{2,A} - A_t^2)\Big\} \\ & + \sigma\Big\{\delta A_t^{2,A} + \beta(1-\sigma)\left[p_{t+1}^f A_{t+1}^{f,h} - \kappa(A_{t+1}^{f,h}) + \delta(A_t^{2,A} - A_{t+1}^{f,h})\right] + \beta\sigma\delta A_t^{2,A}\Big\} \\ & \text{s.t.} \\ & A_t^{2,A} \geq A_t^2 \\ & A_t^{2,A} \geq A_{t+1}^{f,h} \\ & A_t^{2,A} - A_t^2 \geq A_{t+1}^{f,l}.\end{aligned}$$

I know that in the premium equilibrium, the last constraint is binding. The others are not. The Treasury auctions the stock $I_t^2 = \mathcal{I}$. Therefore in equilibrium $A_t^{2,A} = I_t^2 = \mathcal{I}$ and $p_t^{2,A} = \delta + (1-\sigma)\beta\Big\{(1-\sigma)[p_{t+1}^f - \kappa'(A^{f,l})] + \sigma\delta\Big\} + \sigma\beta\delta$.

After adding the auction price, I can now summarise and compare all the prices. In the case of the one-period asset, these are

$$\begin{aligned}p_t^{1,A} &= \delta \\ q_t^n &= (\delta + g) \\ p_t^n &= \beta(1+r_t)(\delta+g).\end{aligned}$$

In the case of the two-period asset, these are

$$\begin{aligned}p_t^{2,A} &= \delta + (1-\sigma)\beta\Big\{(1-\sigma)[p_{t+1}^f - \kappa'(A^{f,l})] + \sigma\delta\Big\} + \sigma\beta\delta \\ q_t^2 &= (1+\beta)(\delta+g) \\ p_t^2 &= \beta(1+r_t)(1+\beta)(\delta+g) \\ p_t^f &= \frac{\sigma}{1-(1-\sigma)\beta(1+r_t)}\beta(1+r_t)(\delta+g).\end{aligned}$$

In addition to the empirical regularity that the on-the-run price is above the off-the-run price, the model explains two other regularities about prices as pointed out in the beginning of this section. First, a detailed analysis by Durham and Perli (2022) showed that when-issued prices carry a premium compared to already issued Treasuries. In their comparison, they include both on- and off-the-run Treasuries. Second, the secondary market price is known to be higher than the auction price (Goldreich (2007), Spindt and Stolz (1992), and Fleming et al. (2022)). See appendix section A.3 for an illustration. The following proposition shows that these two stylised facts also hold in the premium equilibrium.

**Proposition 5.** *In the premium equilibrium*

$$q_t^2 > p_t^2$$
$$q_t^n > p_t^n$$
$$q_t^n > p_t^f.$$

*and*

$$p_t^2 > p_t^{2,A}$$
$$p_t^n > p_t^{1,A}.$$

*Proof.* See appendix A.2.4. □

I will explain the first of the two stylised facts first. It follows from the equations that the spot price (in the case of no fail) is the discounted when-issued price. This can be explained from the seller's perspective. When selling a when-issued contract, the seller receives the when-issued price tomorrow and can deposit it. This income has to cover the spot price he has to pay tomorrow, which in equilibrium is equal to the collateral value he has to build up today. Therefore the spot price in case of no fail is the discounted when-issued price. Compared to off-the-run prices, the when-issued price is even higher, because the off-the-run price takes into account the probability of a settlement fail.

To explain the second stylised fact, the most natural comparison is to compare the auction to the on-the-run price in our model. The auction price is given by the marginal utility of the last unit purchased. The last unit remains in the inventory of the primary dealer and he consumes the coupon $\delta$, except it is sold as an off-the-run asset of a constrained primary dealer. The empirical results of Fleming et al. (2022) suggest that some of the Treasuries remain in the portfolios of the primary dealers until maturity, while the other part is sold. This is consistent with our observation and is reflected in the auction price.

On the other hand, the on-the-run price is determined by the value of the asset to participants in the secondary market. In our case, the buyer is the ultimate owner of the asset, so his marginal utility determines the price. In equilibrium, the spot price is above the coupon rate $\delta$ because the buyer has a higher marginal valuation of the asset. The on-the-run prices are therefore higher than the auction prices.

I summarise the above discussion as follows: The on-the-run prices are driven by the valuation of the buyer of the asset in the secondary market. The auction prices also contains the valuation of the asset if it stays in the inventory of the primary dealer until maturity. The latter valuation is lower than the former. Therefore the primary market prices lie below the secondary market prices. The when issued prices are higher than the spot prices due to the cost of collateralisation.

# 11 Conclusion

I developed a model of the on-the-run phenomenon. I provide a novel explanation using inventory which I also test empirically. The model was then used to discuss broad access to central bank facilities. The

analysis is motivated by the current discussion on how to reform the market (see e.g. Duffie (2023)) and the increase in intermediation and participation by non-bank financial institutions (Eren and Wooldridge (2021)). The latter raises the question of why only a limited number of participants currently have access to central bank facilities.[54] I analysed the implications of providing broad access to a reverse repo or deposit facility. Finally, I added to the literature by extending the model to cover the full life cycle of Treasuries. It includes the auction price, the when-issued price, and the on- and off-the-run prices (all four relevant prices), and can accommodate stylised facts about the relationship between them.

To explain the premium, I endogenised settlement fails and found, consistent with the data, that they are more frequent for off-the-run Treasuries. The probability of a settlement fail is higher for off-the-run Treasuries because they are longer in the market and more dispersed. This leads to a preference for and premium on on-the-run Treasuries. This also explains why the premium is always on the on-the-run Treasury, an aspect that Vayanos and Weill (2008) could not explain. I also test my theory in the data. I show that the premium is higher in times when the probability of failure of off-the-run Treasuries is higher. The probability of fails with off-the-run Treasuries is higher, when the inventories of primary dealers are lower.

Summarising, the on-the-run premium is a symptom that the Treasury market is not frictionless, and inventory risk is higher for off-the-run Treasuries, making them unattractive. Future research should analyse whether the fundamental dynamics described imply a self-fulfilling dynamic that adds to the size of the premium.

The second part focused on the impact of the facility rate when broad access is provided. The result is that an increase in the facility rate leads to a reduction in the cost of trading. The facility complements a trade in the sense that it provides a certain return on liquid funds between trades for those who have access to it. I show that access stimulates trading and prices rise. An increase in the facility rate decreases the premium, which I also confirm in the data. Interestingly, if the facility rate increases, fails increase as more off-the-run Treasuries are traded in contracts promising their delivery but fewer are available. Also, giving access to all institutions is not an act which equalises benefits. Only primary dealers benefit from an increase in the facility rate, not those who gained access.

An interesting way to extend the model would be to take into account more specific characteristics of non-bank financial institutions. This could be, for example, leverage in the case of a hedge fund. In addition, given the current discussions on how to restructure the Treasury market (see e.g. Duffie et al. (2021)), it would be interesting to explore the effects of broad access to a repo facility to complement my analysis.

[54] See for example the FAQ on the repo and the reverse repo facility of the New York FED for eligibility criteria, https://www.newyorkfed.org/markets/repo-agreement-ops-faq and https://www.newyorkfed.org/markets/rrp_faq.

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# A Appendix

## A.1 Other equilibria

Until now I focused on the case where $\lambda_t^2 = \lambda_t^n = \lambda_t^{f,h} = 0$. This means that primary dealers are unconstrained if they still have the full inventory of assets $i$. I showed that I can always find issuance sizes $\mathcal{I}$ where such an equilibrium exists. For the sake of completeness I can also think of equilibria where $\lambda_t^2$, $\lambda_t^n$, and $\lambda_t^{f,h}$ are non-zero. In this case, the quantity of the corresponding assets sold is $\mathcal{I}$. The prices in each of these equilibria are the same as in the premium equilibrium 6 as long as $\lambda_t^{f,l} > 0$, which I need for a positive premium. Since the dynamics and intuition are exactly the same in these equilibria as well, I don't discuss these equilibria in the following, as no additional insights are gained. The only results that would change are the effects of central bank access on quantities. In these other equilibria, there are no quantity effects on assets for which constraints are binding, and only the Lagrange multipliers would change in magnitude.

## A.2 Proofs

### A.2.1 Proof of proposition 1

*Proof.* Suppose it is not the case that $\mathcal{I} < A_{t-1}^2 + A_t^{f,h}$, or $\lambda_t^{f,l} > 0$. Then $\lambda_t^{f,l} = \widetilde{\lambda}_t^{f,l} = 0$. From the first-order conditions of the seller and the buyer and $\omega_t^f = p_t^f$, it follows that $p_t^n = p_t^f = \beta^2(1+r_t)(\delta+g)$. Therefore, there is no premium, $\Delta_t = 0$. The case $\lambda_t^{f,l} = 0$ and $\widetilde{\lambda}_t^{f,l} > 0$ is not possible. If $\lambda_t^{f,l} = 0$, then it must be that $\widetilde{\lambda}_t^{f,l} = \widetilde{\lambda}_t^{f,h} = 0$ given the optimal behavior of the seller which takes into account the profitable amount to be sold by the primary dealer. Also the case where $\lambda_{t+1}^{f,l} > 0$ and $\widetilde{\lambda}_{t+1}^{f,l} > 0$ is not possible. If $\lambda_{t+1}^{f,h} = 0$ and $\lambda_{t+1}^{f,l} > 0$, then $a_t^f > A_t^{f,l}$, and it follows that $\widetilde{\lambda}_{t+1}^{f,l} = 0$. □

### A.2.2 Proof of proposition 2

*Proof.* First, I show that for an equilibrium with trade in all assets to exist, I need $\sigma > \frac{1-\beta(1+r_t)}{\beta(1+r_t)}\frac{\delta}{g}$. In equilibrium $p_t^f = \kappa'(A_t^{f,h}) + \delta = \frac{\sigma}{1-(1-\sigma)\beta(1+r_t)}\beta(1+r_t)(\delta+g)$. For $\kappa'(A_t^{f,h}) > 0$ and $A_t^{f,h} > 0$ I must have that $\frac{\sigma}{1-(1-\sigma)\beta(1+r_t)}\beta(1+r_t)(\delta+g) - \delta > 0$, or $\sigma > \frac{1-\beta(1+r_t)}{\beta(1+r_t)}\frac{\delta}{g}$.

Second, I show that $A_t^2 > A_t^n > A_t^{f,h}$. The first order conditions of the primary dealer with respect to the assets maturing in one period are $p_t^n = \kappa'(A_t^n) + \delta$ and $p_t^f = \kappa'(A_t^f) + \delta$. As $p_t^n > p_t^f$ and $\kappa(A_t^i)$ being a strictly convex function it follows that $A_t^n > A_t^f$. In addition, I can show that $A_t^2 > A_t^n$. The first order condition of the primary dealer with respect to the new two-period asset is:

$$\begin{aligned}
\kappa'(A_t^2) - \beta(1-\sigma)\kappa'(\mathcal{I} - A_t^2) &= p_t^2 - (1+\beta)\delta - \beta(1-\sigma)\kappa'(A_{t+1}^{f,h}) \\
\kappa'(A_t^2) - \beta(1-\sigma)\kappa'(\mathcal{I} - A_t^2) &= (1+\beta)p_t^n - (1+\beta)\delta - \beta(1-\sigma)\kappa'(A_{t+1}^{f,h}) \\
\kappa'(A_t^2) - \beta(1-\sigma)\kappa'(\mathcal{I} - A_t^2) &= (1+\beta)\kappa'(A_t^n) - \beta(1-\sigma)\kappa'(A_{t+1}^{f,h}).
\end{aligned}$$

Further rearrangement yields $\kappa'(A_t^2) - \kappa'(A_t^n) = \beta\kappa'(\mathcal{I} - A_t^2) + \beta[\kappa'(A_t^n) - \kappa'(A_{t+1}^{f,h})] + \beta\sigma[\kappa'(A_{t+1}^{f,h}) - \kappa'(\mathcal{I} - A_t^2)] > 0$. Therefore $A_t^2 > A_t^n$.

Lastly I show that I can always find an $\mathcal{I}$, where the equilibrium exists. In equilibrium $(A^2_{t-1} + A^{f,h}_t) > \mathcal{I} > max(A^2_t, A^n_t, A^{f,h}_t)$. I know that $A^2_t > A^n_t > A^{f,h}_t$. Therefore it follows that in equilibrium $(A^2_{t-1} + A^{f,h}_t) > \mathcal{I} > A^2_t$. From the equilibrium conditions it follows that

$$\begin{aligned} A^n_t &= \kappa'^{-1}\left[\beta(1+r_t)(\delta+g) - \delta\right] \\ A^{f,h}_t &= \kappa'^{-1}\left[\frac{\sigma}{1-(1-\sigma)\beta(1+r_t)}\beta(1+r_t)(\delta+g) - \delta\right] \\ \kappa'(A^2_{t-1}) &= W + \beta(1-\sigma)\kappa'(\mathcal{I} - A^2_{t-1}) \end{aligned}$$

where $W \equiv \left[(1+\beta) - \beta(1-\sigma)\frac{\sigma}{1-(1-\sigma)\beta(1+r_t)}\right]\beta(1+r_t)(\delta+g) - (1+\beta\sigma)\delta$.

The left-hand side of the third equation, $f_1(x) = \kappa'(x)$, is a strictly increasing function with $x \in [0, \mathcal{I}]$ and $min(f_1(x)) = 0$ and $max(f_1(x)) = \kappa'(\mathcal{I})$. The right-hand side of the third equation, $f_2(x) = W + \beta(1-\sigma)\kappa'(\mathcal{I} - x)$, is a strictly decreasing function with $x \in [0, \mathcal{I}]$ and $min(f_2(x)) = W$ and $max(f_2(x)) = W + \beta(1-\sigma)\kappa'(\mathcal{I})$. Therefore for the equilibrium to exist it must be that $\kappa'(\mathcal{I}) > W$, or $\mathcal{I} > \kappa'^{-1}(W)$. By continuity it then follows that $\mathcal{I} > A^2_{t-1}$. As $\mathcal{I} - A^2_{t-1}(\mathcal{I})$ is a continuous function (as $\kappa'(A^i_t)$ and $\kappa'^{-1}(A^i_t)$ are both continuous) with minimal value 0 for $\mathcal{I} = \kappa'^{-1}(W)$ I can always find an $\mathcal{I}$ where $\mathcal{I} - A^2_{t-1}(\mathcal{I}) < A^{f,h}_t$ for any $A^{f,h}_t > 0$. □

### A.2.3 Proof of proposition 4

*Proof.* First, I know that in equilibrium $f^n_t = 0$ and $f^f_t = \frac{(1-\sigma)p^f_t(a^f_t - I^f_t)}{p^f_t a^f_t} = (1-\sigma)\left(1 - \frac{\mathcal{I} - a^2_t}{a^f_t}\right)$. Therefore, trivially, $f^f_t > 0 = f^n_t$. Second, from comparing $p^n_t = \beta(1+r_t)(\delta+g)$ and $p^f_t = \frac{\sigma}{1-(1-\sigma)\beta(1+r_t)}\beta(1+r_t)\beta(\delta+g)$ it follows immediately that $p^n_t > p^f_t$. Third, $p^n_t > p^f_t$ also implies that $A^n_t > A^{f,h}_t$ as $A^n_t = \kappa'^{-1}(p^n_t - \delta)$ and $A^{f,h}_t = \kappa'^{-1}(p^f_t - \delta)$. Lastly, as $\lambda^{f,h}_t = 0$ and $\lambda^{f,l}_t > 0$ it follows that $A^{f,h}_t > A^{f,l}_t$. □

### A.2.4 Proof of proposition 5

*Proof.* First, as $\beta(1+r_t) > 0$, it follows that $q^2_t > p^2_t$ and $q^n_t > p^n_t$. Also, from proposition 4, I know that $p^n_t > p^f_t$ and therefore $q^n_t > p^f_t$. Second, from the first order condition of the primary dealer (see section 4.1) I know that $p^2_t = \kappa'(A^2_t) + \delta + \beta\delta + \beta(1-\sigma)\lambda^f_{t+1}$ where $\lambda^f_{t+1} = p^f_{t+1} - \kappa'(A^{f,l}) - \delta$. It follows immediately that $p^2_t > p^{2,A}_t$. Also from the first order condition of the primary dealer, I know that $p^n_t = \kappa'(A^n_t) + \delta$. It follows immediately that $p^n_t > p^{1,A}_t$. □

### A.2.5 Proof of corollary 1

*Proof.* I assume an unanticipated permanent increase in the interest rate that occurs at the beginning of the contract market in $t = \widetilde{t} - 1$. The stock of available off-the-run assets in the next spot market is given, and it can reach a new steady state only after a period.

First, I show that an increase in the facility rate $r_t$ raises all prices. I know that in equilibrium, prices are given by $p^n_t = \beta(1+r_t)(\delta+g)$, $p^2_t = \beta(1+r_t)(1+\beta)(\delta+g)$, $p^f_t = \frac{\sigma}{1-(1-\sigma)\beta(1+r_t)}\beta(1+r_t)(\delta+g)$. It

follows that

$$\frac{dp_t^n}{dr_{\widetilde{t}-1}} = \beta(\delta+g) > 0$$
$$\frac{dp_t^2}{dr_{\widetilde{t}-1}} = \beta(1+\beta)(\delta+g) > 0$$
$$\frac{dp_t^f}{dr_{\widetilde{t}-1}} = \left[\frac{\sigma}{1-(1-\sigma)\beta(1+r_t)} + \frac{\sigma(1-\sigma)\beta(1+r_t)}{\left[1-(1-\sigma)\beta(1+r_t)\right]^2}\right]\beta(\delta+g) > 0$$

for all $t \geq \widetilde{t}$.

Second, I show that the on-the-run premium decreases given an increase in $r_t$. In equilibrium $\frac{dp_t^n}{dr_{\widetilde{t}-1}} < \frac{dp_t^f}{dr_{\widetilde{t}-1}}$ if $\frac{\sigma}{1-(1-\sigma)\beta(1+r_t)} + \frac{\sigma(1-\sigma)\beta(1+r_t)}{[1-(1-\sigma)\beta(1+r_t)]^2} > 1$ which is the case if $\sigma > \frac{[1-\beta(1+r_t)]^2}{[\beta(1+r_t)]^2} \approx 0$.[55] The premium is given by $\Delta_t = p_t^n - p_t^f$. Therefore $\frac{d\Delta_t}{dr_{\widetilde{t}-1}} = \frac{dp_t^n}{dr_{\widetilde{t}-1}} - \frac{dp_t^f}{dr_{\widetilde{t}-1}} < 0 \; \forall t \geq \widetilde{t}$.

Third, I discuss the impact on the quantities. I know that in equilibrium quantities are determined by $\kappa'(A_t^n) = p_t^n - \delta$, $\kappa'(A_t^2) - \beta(1-\sigma)\kappa'(I_t^2 - A_t^2) = p_t^2 - \beta(1-\sigma)p_{t+1}^f - (1+\beta\sigma)\delta$, and $\kappa'(A_t^{f,h}) = p_t^f - \delta$. It follows that

$$\frac{d\kappa'(A_t^n)}{dr_{\widetilde{t}-1}} = \frac{dp_t^n}{dr_{\widetilde{t}-1}} > 0$$
$$\frac{d\kappa'(A_t^{f,h})}{dr_{\widetilde{t}-1}} = \frac{dp_t^f}{dr_{\widetilde{t}-1}} > 0$$

for all $t \geq \widetilde{t}$. Therefore $\frac{dA_t^n}{dr_{\widetilde{t}-1}} > 0$ and $\frac{dA_t^{f,h}}{dr_{\widetilde{t}-1}} > 0$ for all $t \geq \widetilde{t}$. Also

$$\frac{d\kappa'(A_t^2)}{dr_{\widetilde{t}-1}} - \beta(1-\sigma)\frac{d\kappa'(I_t^2 - A_t^2)}{dr_{\widetilde{t}-1}} = \frac{dp_t^2}{dr_{\widetilde{t}-1}} - \beta(1-\sigma)\frac{dp_{t+1}^f}{dr_{\widetilde{t}-1}}.$$

It follows that $\frac{dA_t^2}{dr_{\widetilde{t}-1}} > 0$ for all $t \geq \widetilde{t}$ iff $\frac{dp_t^2}{dr_{\widetilde{t}-1}} > \beta(1-\sigma)\frac{dp_{t+1}^f}{dr_{\widetilde{t}-1}}$. $\frac{dp_t^2}{dr_{\widetilde{t}-1}} > \beta(1-\sigma)\frac{dp_{t+1}^f}{dr_{\widetilde{t}-1}}$ if $\sigma > \widetilde{\sigma}$.

The value of $\widetilde{\sigma}$ is derived as follows:

$\frac{dp_t^2}{dr_{\widetilde{t}-1}} > \beta(1-\sigma)\frac{dp_{t+1}^f}{dr_{\widetilde{t}-1}}$ iff $(1+\beta) > \beta(1-\sigma)\left[\frac{\sigma}{1-(1-\sigma)\beta(1+r_t)} + \frac{\sigma(1-\sigma)\beta(1+r_t)}{[1-(1-\sigma)\beta(1+r_t)]^2}\right]$, or

$(1-\beta(1+r_t))^2 > \frac{\sigma}{(1-\sigma)}\left[(1-\sigma)\left(\frac{\beta}{(1+\beta)} + (\beta(1+r_t))^2\right) - 1\right]$, where $(1-\beta(1+r_t))^2 \approx 0$. There are two values of $\sigma$ which solve the equation $(1-\beta(1+r_t))^2 = \frac{\sigma}{(1-\sigma)}\left[(1-\sigma)\left(\frac{\beta}{(1+\beta)} + (\beta(1+r_t))^2\right) - 1\right]$. Below the lower root and above the upper root the right-hand side is lower than the left-hand side. I define $\widetilde{\sigma}$ as the larger root. Note that using empirically reasonable values, it can only for a limited range of small values of $\sigma$ be that $\frac{dp_t^2}{dr_{\widetilde{t}-1}} > \frac{dp_{t+1}^f}{dr_{\widetilde{t}-1}}$ is not true. For example if $\beta = 0.96$ and $r_{\widetilde{t}-1} = 0.01$, then only for $\sigma \in (0.003, 0.25)$ it follows that $\frac{dp_t^2}{dr_{\widetilde{t}-1}} < \frac{dp_{t+1}^f}{dr_{\widetilde{t}-1}}$.

Next I look at the quantity of off-the-run assets traded by constrained dealers, which is given by $A_t^{f,l} = \mathcal{I} - A_{t-1}^2$. It follows that $\frac{dA_t^{f,l}}{dr_{\widetilde{t}-1}} = 0$ for $t = \widetilde{t}$ because $\frac{dA_{\widetilde{t}-1}^2}{dr_{\widetilde{t}-1}} = 0$. For all $t > \widetilde{t}$ it follows

[55] Let me define $k \equiv (1-\sigma)\beta(1+r_t)$. It follows: $\frac{\sigma}{1-k} + \frac{\sigma k}{(1-k)^2} > 1$, or $\sigma > (1-k)^2$. Further rearranging yields $0 > 1 - 2\beta(1+r_t) + (1-\sigma)\left[\beta(1+r_t)\right]^2$, or $\sigma > \frac{[1-\beta(1+r_t)]^2}{[\beta(1+r_t)]^2} \approx 0$.

that $\frac{dA_t^{f,l}}{dr_{\widetilde{t}-1}} < 0$ if $\frac{dA_t^2}{dr_{\widetilde{t}-1}} > 0$ which is true if $\sigma > \widetilde{\sigma}$. From the above results, it directly follows that $\frac{d(A_t^2+A_t^n+\sigma A_t^{f,h}+(1-\sigma)A_t^{f,l})}{dr_{\widetilde{t}-1}} > 0$ for $t = \widetilde{t}$ and $\forall t > \widetilde{t}$ if $\sigma > \widetilde{\sigma}$. Decreases in the trading of off-the-run assets by constrained primary dealers are due to equivalent increases in the trading of the asset in the on-the-run period.

Lastly, the off-the-run settlement failure rate is given by $f_t^f = \frac{(1-\sigma)p_t^f(a_t^f - I_t^f)}{p_t^f a_t^f} = (1-\sigma)\left(1 - \frac{\mathcal{I}-a_{t-1}^2}{a_t^f}\right)$. It follows that $\frac{df_t^f}{dr_{\widetilde{t}-1}} > 0$ for $t = \widetilde{t}$ because $\frac{dp_t^f}{dr_{\widetilde{t}-1}} > 0$ for $t \geq \widetilde{t}$ and $\frac{\partial A_{t-1}^2}{\partial r_{\widetilde{t}-1}} = \frac{\partial a_{t-1}^2}{\partial r_{\widetilde{t}-1}} = 0$. For $\forall t > \widetilde{t}$ it follows that $\frac{df_t^f}{dr_{\widetilde{t}-1}} > 0$ iff $\frac{\partial A_{t-1}^2}{\partial r_{\widetilde{t}-1}} = \frac{\partial a_{t-1}^2}{\partial r_{\widetilde{t}-1}} > 0$ which is the case if $\sigma > \widetilde{\sigma}$. □

### A.2.6 Proof of corollary 2

*Proof.* I look at the impact of an increase in the interest rate $r_t$ (see details below) on the lifetime values of the buyer, the seller, and the primary dealer. There are two groups of primary dealers: One group of primary dealers has an inventory of $\mathcal{I}$ off-the-run assets and another has one of $\mathcal{I} - A_t^2$. I take the lifetime value of both groups and average according to their share in the population. To ease notation, let me define $V_{\widetilde{t}-1}^{D,a} \equiv (1-\sigma)V_{\widetilde{t}-1}^D(\mathcal{I} - A_t^2) + \sigma V_{\widetilde{t}-1}^D(\mathcal{I})$. The lifetime values are given by

$$
\begin{aligned}
V_{\widetilde{t}-1}^b &= \sum_i \left[\beta(\delta+g) - \beta q_{\widetilde{t}-1}^i\right] a_{\widetilde{t}}^i + \beta^2(\delta+g)a_{\widetilde{t}}^2 - \beta(1-\sigma)[(\delta+g) - \omega_{\widetilde{t}-1}^f](a_{\widetilde{t}}^f - I_{\widetilde{t}}^f) + \beta V_{\widetilde{t}}^b \\
V_{\widetilde{t}-1}^s &= \sum_i \left\{-\omega_{\widetilde{t}-1}^i + \beta\left[q_{\widetilde{t}-1}^i - p_{\widetilde{t}}^i + \omega_{\widetilde{t}-1}^i\right](1+r_t)a_{\widetilde{t}}^i\right\} - \beta(1-\sigma)[\omega_{\widetilde{t}-1}^f - p_{\widetilde{t}}^f](1+r_t)(a_{\widetilde{t}}^f - I_{\widetilde{t}}^n) \\
&\quad + \beta V_{\widetilde{t}-1}^s \\
V_{\widetilde{t}-1}^{D,a} &= \beta\{(1-\sigma)^2[p_{\widetilde{t}}^f(I_{\widetilde{t}-1}^2 - A_{\widetilde{t}-1}^2) - \kappa(I_{\widetilde{t}-1}^2 - A_{\widetilde{t}-1}^2) - \delta(I_{\widetilde{t}-1}^2 - A_{\widetilde{t}-1}^2)] \\
&\quad + (1-\sigma)\sigma[p_{\widetilde{t}}^f A_{\widetilde{t}}^{f,h} - \kappa(A_{\widetilde{t}}^{f,h}) - \delta A_{\widetilde{t}}^{f,h}] + (1-\sigma)\sum_{j\in\{n,2\}}\left[p_{\widetilde{t}}^j A_{\widetilde{t}}^j - \kappa(A_{\widetilde{t}}^j) - \delta A_{\widetilde{t}}^j\right] \\
&\quad + \delta(I_{\widetilde{t}}^n + I_{\widetilde{t}}^2 + I_{\widetilde{t}-1}^2 - (1-\sigma)A_{\widetilde{t}-1}^2)\} + \beta V_{\widetilde{t}}^{D,a}.
\end{aligned}
$$

I assume that there is an unanticipated permanent increase in the interest rate at the beginning of the contract market in $t = \widetilde{t} - 1$. The stock of available off-the-run assets in the next spot market is given and can reach a new steady state only after a period. New steady state values have no time index. I can simplify the buyer's and seller's profit using the first-order conditions in equilibrium. This yields

$$
\begin{aligned}
V_{\widetilde{t}-1}^b &= \beta(1-\sigma)[(\delta+g) - p_{\widetilde{t}}^f]I_{\widetilde{t}}^f + \frac{\beta^2}{1-\beta}(1-\sigma)\left[(\delta+g) - p^f\right]I^f \\
V_{\widetilde{t}-1}^s &= 0.
\end{aligned}
$$

The derivatives are

$$
\begin{aligned}
\frac{dV_{\widetilde{t}-1}^b}{dr_{\widetilde{t}-1}} &= -\beta(1-\sigma)\frac{dp_{\widetilde{t}}^f}{dr_{\widetilde{t}-1}}(\mathcal{I} - a_{\widetilde{t}-1}^2) - \frac{\beta^2}{1-\beta}(1-\sigma)\left\{\frac{dp^f}{dr_{\widetilde{t}-1}}(\mathcal{I}-a^2) + \left[(\delta+g) - p^f\right]\frac{da^2}{dr_{\widetilde{t}-1}}\right\} \\
\frac{dV_{\widetilde{t}-1}^d}{dr_{\widetilde{t}-1}} &= 0
\end{aligned}
$$

$$\frac{dV^{D,a}_{\widetilde{t}-1}}{dr_{\widetilde{t}-1}} = \beta\Bigg\{(1-\sigma)^2\frac{dp^f_{\widetilde{t}}}{dr_{\widetilde{t}-1}}(\mathcal{I}-A^2_{\widetilde{t}-1}) + (1-\sigma)\sigma\frac{d[\kappa'(A^{f,h}_{\widetilde{t}})A^{f,h}_{\widetilde{t}} - \kappa(A^{f,h}_{\widetilde{t}})]}{dr_{\widetilde{t}-1}}$$
$$+ (1-\sigma)\sum_{j\in\{n,2\}}\frac{d[\kappa'(A^j_{\widetilde{t}})A^j_{\widetilde{t}} - \kappa(A^j_{\widetilde{t}})]}{dr_{\widetilde{t}-1}}\Bigg\}$$
$$+ \frac{\beta^2}{1-\beta}\Bigg\{(1-\sigma)^2\frac{d[\kappa'(\mathcal{I}-A^2)(\mathcal{I}-A^2) - \kappa(\mathcal{I}-A^2) - \beta(1-\sigma)\kappa'(\mathcal{I}-A^2_{\widetilde{t}-1})\mathcal{I}]}{dr_{\widetilde{t}-1}}$$
$$+ (1-\sigma)\sigma\frac{d[\kappa'(A^{f,h})A^{f,h} - \kappa(A^{f,h})]}{dr_{\widetilde{t}-1}} + (1-\sigma)\sum_{j\in\{n,2\}}\frac{d[\kappa'(A^j)A^j - \kappa(A^j)]}{dr_{\widetilde{t}-1}}\Bigg\}.$$

I can simplify the last equation. This yields

$$\frac{dV^{D,a}_{\widetilde{t}-1}}{dr_{\widetilde{t}-1}} = \beta(1-\sigma)^2\frac{dp^f_{\widetilde{t}}}{dr_{\widetilde{t}-1}}(\mathcal{I}-A^2_{\widetilde{t}-1}) + \frac{\beta}{1-\beta}\Bigg\{\beta(1-\sigma)^2\kappa''(\mathcal{I}-A^2)A^2\frac{\partial A^2}{\partial r_{\widetilde{t}-1}}$$
$$+ (1-\sigma)\sigma\kappa''(A^{f,h})A^{f,h}\frac{\partial A^{f,h}}{\partial r_{\widetilde{t}-1}} + (1-\sigma)\sum_{j\in\{n,2\}}\kappa''(A^j)A^j\frac{\partial A^j}{\partial r_{\widetilde{t}-1}}\Bigg\}.$$

Regarding the profits of the buyer, note that $(\delta+g)-p^f>0$. If $\frac{\partial A^2}{\partial r_{\widetilde{t}-1}} = \frac{\partial a^2}{\partial r_{\widetilde{t}-1}} > 0$, which is the case if $\sigma > \widetilde{\sigma}$, then $\frac{dV^b_{\widetilde{t}-1}}{dr_{\widetilde{t}-1}} < 0$, $\frac{dV^s_{\widetilde{t}-1}}{dr_{\widetilde{t}-1}} = 0$, and $\frac{dV^{D,a}_{\widetilde{t}-1}}{dr_{\widetilde{t}-1}} > 0$. □

### A.2.7 Proof of corollary 4

*Proof.* I analyse the impact if sellers without access gain access, meaning that their interest rate unanticipated permanently increases from $r^m_t$ to $r_t$ in the second subperiod in $t = \widetilde{t}-1$. This implies that all sellers now trade at the same prices $p^{j,a}_t$ and quantities $A^{j,a}_t$ for $j = 2$ and $n$, i.e. all the newly issued assets. Therefore the spot prices and quantities of all on-the-run assets traded by sellers who gain access increase as $p^{2,a}_t > p^{2,na}_t$, $p^{n,a}_t > p^{n,na}_t$, $A^{2,a}_t > A^{2,na}_t$, $A^{n,a}_t > A^{n,na}_t$, while the prices and quantities for the ones who already had access stay the same.

Following the same reasoning, in the first period after the shock in $t = \widetilde{t}$, the same result also holds for the off-the-run prices $p^f_t$ as well as off-the-run quantities if no inventory constraint binds, i.e. for $A^{f,h}_t$. For this result, it is important to note that the probabilities of finding the assets have not yet changed because inventories are predetermined from the previous period. Predetermined inventories also imply that the off-the-run quantities sold if inventory constraints bind, $A^{f,l}_t = \mathcal{I} - A^2_{t-1}$, stay the same in this first period. The on-the-run premium initially, i.e. in $t = \widetilde{t}$, decreases following the same reasoning as in corollary 2.

To analyse the off-the-run prices and quantities traded in the periods $t > \widetilde{t}$, I must take into account how the probabilities to find the assets change after the first period after the shock. Otherwise, the reasoning is analogous. I assume that settlement fails still occur, so that there is still a positive premium (as in section 8.1). The probability to find the assets is for all sellers $\sigma$. This is weakly lower than beforehand. If already before access $\mathbb{P}^a = \mathbb{P}^{na} = \sigma$, then the effects are the same as in the first period after the shock. Otherwise, the effects on the prices, the premium, and the quantities traded are ambiguous. This is because, as implied by the equations for $p^f_t$ and $A^{f,h}_t$, the effects of the interest rate

change and the probability change work in opposite directions. □

## A.3 Figures

The following graphs show the stylised facts and irregularities that are discussed in the paper.

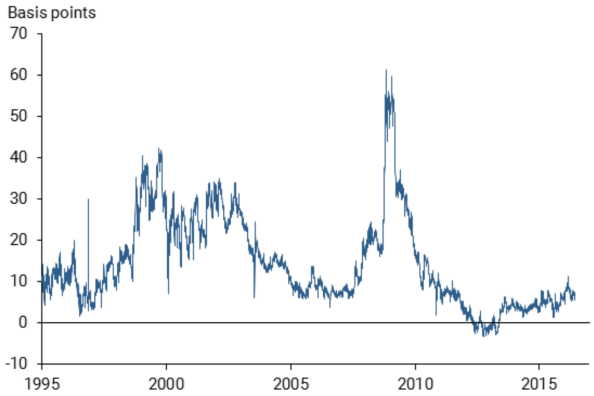



Figure 10: On-the-run premium on 10 year bonds

Figure 10 is taken from Christensen et al. (2017) and shows the on-the-run premium on 10 year Treasury bonds. The on-the-run yield is subtracted from the par yield of seasoned bonds.

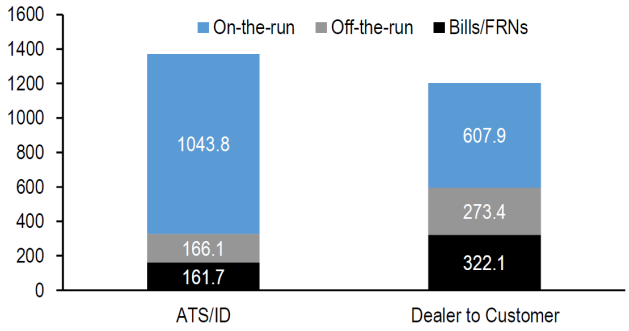



Figure 11: Treasury trading volumes by counterparty in billion USD

Figure 11 shows Treasury trading volumes by counterparty. ATS stands for automated trading system and ID for interdealer (U.S. Department of the Treasury (2022)). I can see that the trading volume in on-the-run Treasuries is much higher than in off-the-run Treasuries, regardless of the counterparty. This is also shown in the next chart.

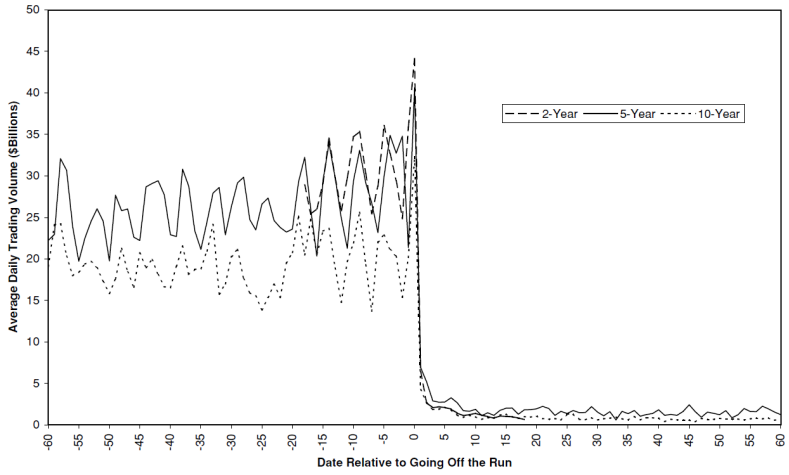



Figure 12: Trading volume for on-the-run and off-the-run Treasuries

Figure 12 shows the average trading volume for on- and off-the-run 2 year, 5 year, and 10 year Treasury notes relative to the date they went off-the-run (Barclay et al. (2006)).

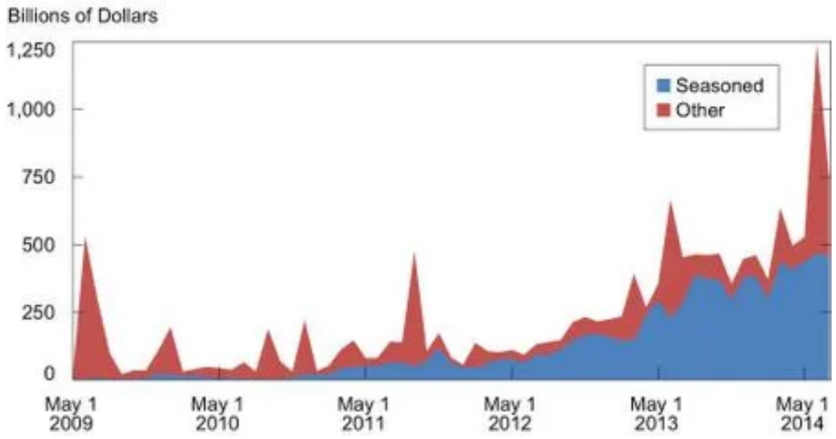



Figure 13: Fails in seasoned and other Treasuries

Figure 13, provided by Fleming et al. (2014), plots the cumulative gross fails by month in seasoned Treasuries and all other Treasuries (including on-the-run Treasuries). Seasoned Treasuries are Treasuries issued more than 180 days ago. Fleming et al. (2014) point out that there has been a steady increase in the number of fails of seasoned Treasuries over the past few years.

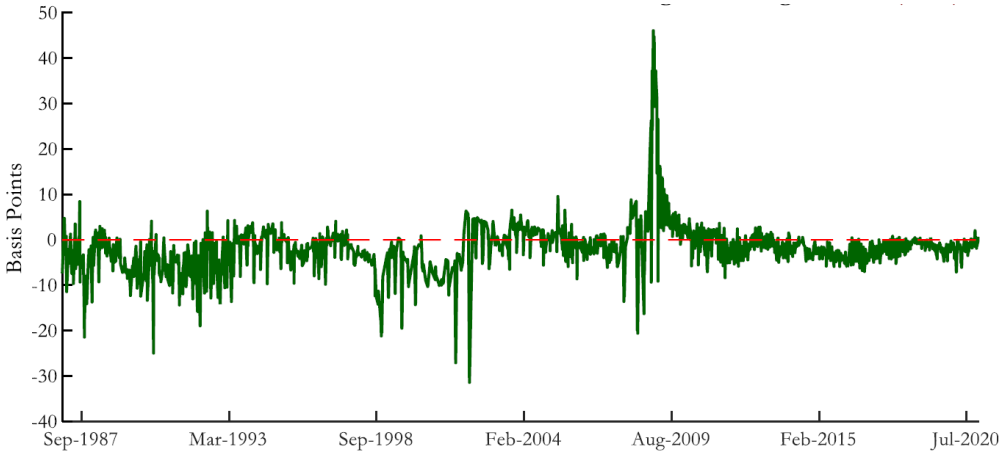



Figure 14: When-issued premium

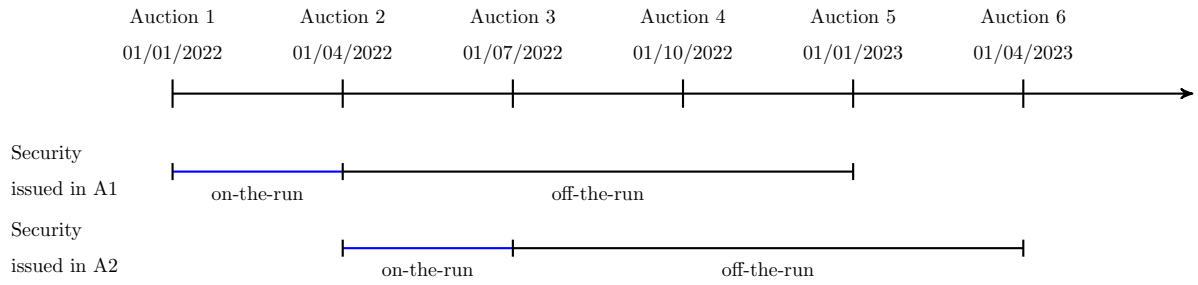
The conditional when-issued premium, shown in figure 14, was measured by Durham and Perli (2022). The premium is computed by regressing the difference between the actual and the fitted yield on control variables and a dummy variable indicating whether the Treasury is when-issued.

| Time of Day | No. of Auctions | Underpricing (relative to when-issued transactions) | Standard Error | Underpricing (relative to when-issued bid quote) | Standard Error |
|---|---|---|---|---|---|
| 12:50 PM–1:00 PM | 178 | 0.32*** | 0.12 | 0.20** | 0.12 |
| 12:30 PM–1:00 PM | 178 | 0.40*** | 0.12 | 0.29*** | 0.12 |
| 1:00 PM–1:30 PM | 176 | 0.32*** | 0.10 | 0.15* | 0.10 |

Figure 15: Underpricing

Fleming et al. (2022) document in their paper that Treasury dealers appear to be compensated for taking inventory risks at the auction by price increases in subsequent weeks. This is evidence that auction prices are lower than secondary market prices after the auction. A direct comparison of primary and secondary market prices at the time of the auction has been done by Goldreich (2007) and Spindt and Stolz (1992), among others. Both show that the primary market price is lower than the secondary market price of the same security.[56] The table above from Goldreich (2007) compares the auction yields with the when-issued yields in the minutes before and after the auction. As Goldreich (2007) explains, for example, an underpricing of 0.32 basis points (first row, third column) is equivalent to 1.3 cents per 100$.

[56] Underpricing is also an observed phenomenon in Initial Public Offerings (IPO's). See for example Chambers and Dimson (2009).

## A.4 On-the-run cycle

The following figure illustrates the on-the-run cycle. An auction is held every quarter. The newly issued security is on-the-run until the next auction of assets with the same maturity. It then becomes off-the-run.