ESTIMATING THE TERM STRUCTURE OF INTEREST RATES: THE SWISS CASE

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Abstract

Parametric estimation approaches are widely by central banks as they produce smooth term structures with relatively few parameters. In the paper I implement the Nelson and Siegel (1987) model for Switzerland. The estimations use daily observations of Swiss government bonds from January 1994 to July 1998. To overcome the lack of suffient data in the very short run, the 1-month and 1-year Euromarket rate are added. The knowledge of the dependencies of the term structure from the possible parameter constellations is used to calibrate the model for the Swiss market. The results show that the parameters are stable over time. The smooth shape and the stablility over time make it a valuable tool for monetary policy.

Keywords: Term structure of interest rates, Interpolation
JEL code: E43

1 Introduction

This paper estimates the Swiss term structure using non-callable government bonds. As zero coupon bonds are not traded on the Swiss government bond market, we cannot derive the spot rate curve directly. I therefore use the Nelson and Siegel (1987) procedure to derive the term structure from coupon-bearing bonds for daily observations of Swiss government bonds over the period from January 1994 to July 1998. The Nelson/Siegel model approximates the shape of the term structure with a sequence of exponential terms. The aim is to get a smooth term structure that is flexible enough to replicate the shape of the actual term structure. A smooth term structure

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implies stable forward rates, a desirable property for the interpretation in monetary policy as well as for pricing interest rate derivatives. The main emphasis of the paper is placed on the numerical implementation for the Swiss market. I show how on economic grounds one can set up boundaries and restrictions that ensure convergence of the optimization process. Together with the starting values, these boundaries are crucial to guarantee an accurate term structure estimation on all 1176 observation days.

The remainder of the paper is organized as follows. Section 2 reviews the literature and the experience from other countries. Section 3 describes the basic concepts which are important for estimating and interpreting the term structure. The Nelson/Siegel model is presented in section 4 followed by an description of the different steps to estimate the model in section 5. Section 6 presents the empirical results of the application to daily prices of Swiss government bonds from January 24, 1994 to July 31, 1998. Section 7 summarizes the paper and makes concluding remarks.

2 The Experience with Term Structure Models Based on Bond Data

The method by Nelson and Siegel (1987) and the extended version by Svensson (1994) are widely used by central banks. Among others, the Bank of England, the Deutsche Bundesbank, the Österreichische Nationalbank and the Banque de France make use of these estimation techniques. Nelson and Siegel (1987) propose the model to fit a term structure to U.S. Treasury bill yields. They use 37 monthly samples from January 1981 through October 1983. The objective of their empirical work was to find a parsimonious model that is adequate to describe the relationship between interest rates and time to maturity. Svensson (1994) applies his extension to weekly data from Sweden for the period from May 1992 to June 1994. He demonstrates the estimation and the interpretation of the term structure as an indicator of market expectations of future short rates, monetary policy, inflation rates and currency depreciation rates. Both methods have become popular as they reconcile the following three characteristics:

- Sufficient flexibility to reflect the important and typical patterns of the observed market data.
- Relatively robust against disturbances from individual observations.
- Applicable with only a few observations.

In November 1994 the Bank of England adopted the Svensson (1994) technique for estimating yield curves from government bonds.[1] The in-

[1] Before using the Svensson approach, the Bank of England fitted a cubic spline through yields to maturity by minimizing the squared differences between observed and fitted

troduction and the institutional adaptations are summarized by Breedon (1995) in the Quarterly Report of the Bank of England. Deacon and Derry (1994a; 1994b)[2] report on the evaluation of different alternatives. In the United Kingdom investors who face a marginal income tax rate but no taxation on capital gains form a large proportion of the market so that low coupon bonds carry a price premium. The Bank of England therefore introduces two further parameters to capture these tax effects.[3]

The Deutsche Bundesbank followed in fall 1997 and introduced the Svensson (1994) approach in their publications. Schich (1997) describes in detail the numerical specifications chosen by the Bundesbank. It replaced the yield curve approximated by a linear-logarithmic regression analysis that is described in the appendix of the "Monatsberichte der Deutschen Bundesbank" (1983; 1991)[4]. The Bundesbank announced the change in method in the Monatsbericht (1997). The same article graphically illustrates how the new approach allows for more flexibility and enumerates the advantages. For example, (a) implied forward rates can be directly derived from the given spot rates, (b) expectations can be analyzed with better accuracy and (c) as

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yields. The estimation procedure also included adjustments for tax effects by modelling the dependence of the yield from coupon and maturity. Steeley (1991) criticizes the imprecise estimation results of conventional spline estimations.

[2] The working paper (1994b) describes the whole research process. The following list of models has been investigated by the Bank of England: McCulloch (1975), Schaefer (1981), Nelson and Siegel (1987) and Svensson (1994). The Quarterly Report (1994a) provides a survey of the main results.

[3] The parameters to adjust for tax effects are described by Cooper and Steeley (1996): (a) In the United Kingdom bonds trade ex-dividend during a certain period of the year. The preferential tax treatment of capital gains compared to coupon income results in a price premium for ex-dividend traded bonds. (b) Another source for a possible price premium is the declariation of some bonds as Free Of Tax for Residents Abroad (FOTRA).

[4] The approach was based on government debt (Bund, Bahn and Post). Bonds were sorted according to time to maturity. Then an average group rate $\overline{R}$ was calculated for each time to maturity segment i. Before 1983, the influence of the level of the coupons on the price was neglected, as test calculations in 1978 have revealed no systematic relationship between nominal return and yield to maturity (coupon effect). The estimated equation was

$$r\left(\overline{R}_i\right) = \beta_0 + \beta_1\,\overline{T}_i + \beta_2\,\ln\left(\overline{T}_i\right),$$

where $\overline{T}_i$ denotes the average time to maturity for a specific time to maturity segment i. After 1978 interest rate movements on the German market increased and the coupon effect got more important. This induced the Bundesbank in 1983 to augment the above regression equation by two terms to account for the coupon effect. The average nominal coupon rate was added as a second explaining factor of the term structure equation. The resulting equation became

$$r\left(\overline{R}_i\right) = \beta_0 + \beta_1\,\overline{T}_i + \beta_2\,\ln\left(\overline{T}_i\right) + \beta_3\,\overline{C}_i + \beta_4\,\ln\left(\overline{C}_i\right)$$

with two additional terms depending on the average nominal coupon rate $\overline{C}$ in the time to maturity segment i. Beginning in 1983 the extended version had been estimated and the published yield term structures had been corrected by the coupon effects.

long-term interest rates converge to a constant, the term structure models of Nelson/Siegel and Svensson produce plausible extrapolations. The former linear-logarithmic regression approach contained terms that are linearly related to time to maturity and hence extrapolation may generate negative or infinitely high interest rates.

The empirical investigation on the Austrian government bond market by Brandner and Jaeger (1992) compares the Nelson/Siegel model and the approach by the Österreichische Kontrollbank.[5] The conclusion is that the Nelson/Siegel model performs better. Geyer and Mader (1999) estimate the Nelson/Siegel model for the Österreichische Nationalbank. Before applying the model to the data set of government bond yields the authors exclude outliers.[6] Besides Austria, they analyze the term structure for Germany, U.K., U.S.A. and Japan over the period from 1993 to 1998. They find, that the Svensson model does not improve the estimation results.

Ricart and Sicsic (1995) apply the Nelson/Siegel model to French government bonds from 1980-95. In line with Geyer and Mader (1999), they prefer the parsimonious Nelson/Siegel model to the extension by Svensson because at the short end of the term structure there are not enough bond yields available to accurately estimate the extended version. Using these Nelson/Siegel term structure estimates for France, Jondeau and Ricart (1997) evaluate the information content of the term structure regarding future changes in interest rates and changes in inflation.

For Switzerland, Heller (1997) estimates the term structure for April 23, 1997 using the Nelson and Siegel (1987) method. He illustrates the power of the procedure for monetary purposes and shows how information about inflation expectations can be extracted using the expectations hypothesis and the Fisher equation. Sommer (1999) employs the Nelson/Siegel procedure to estimate the term structure for different rating classes.[7] Tobler (1999) proposes a two-step procedure that combines the Nelson/Siegel approximation with basis spline approximation for shorter maturities.

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[5] The approach by the Österreichische Kontrollbank is similar to the former approach of the Deutsche Bundesbank used until 1983. However, the dependent variable is the logarithm of the yields, $\ln(R)$.

$$\ln(R) = \beta_0 + \beta_1 \ln(T),$$

where T is again the time to maturity. For a detailed discussion see Pichler (1998).

[6] Geyer and Mader (1999): "First, a smoothed curve is fitted to the observed yields using Cleveland's (1979) smoothing algorithm. Second, yields of bonds that are 'too far' from the smoothed curve are considered to be outliers and excluded. The decision regarding outliers is based on the interquartile range and a parameter whose value has been fixed on the basis of experiments."

[7] Previous studies of Beer (1990) and Deppner (1992) compared different (non)linear regression and optimization methods. However, both studies did not analyze the Nelson-Siegel method.

Most studies use government bond prices, since these are regarded as being free of default risk. However, when comparing the results for different countries the following problems should be kept in mind.

- The estimation results depend heavily on the availability of bonds and how they are divided along the maturity range.[8] Callable and convertible bond prices reflect embedded options and are not comparable to standard coupon bond prices. Therefore, these bonds should be excluded from the estimations. Furthermore, the liquidity of the market determines the quality of the data.

- Effects of taxation can distort bond prices substantially. Research on tax effects in estimating the term structure of interest rates goes back to McCulloch (1971; 1975). In many countries coupon payments and capital gains are treated differently and possibly not in the same manner for all market participants.

- To get the economically correct yields to maturity, accrued interest must be added to quoted bond prices. But as Cooper and Steeley (1996) point out in their study on the G7 nations,[9] accrued interest is accounted for differently across countries.

3 The Relationship between Spot Rates, Forward Rates and Bond Yields

3.1 Spot and Forward Rates

The models implemented in this study are formulated in continuous time and thus all relationships in this section are also expressed in continuous time. This simplifies the notation, as multiplications in discrete time become additive in continuous time and cross products drop out.

For each time to maturity t, there exists a unique underlying interest rate, the spot rate $r(t)$. It is the appropriate interest rate to discount a payment due in t years. If the spot rate is a continuously compounded annual rate, the present value of a payment F in t years equals

$$P = \exp\left[-t\, r(t)\right]\ F_t. \tag{1}$$

The spot rates for riskless assets for different times to maturity form the term structure. These rates are independent of e.g. taxation rules or default risk.

[8] In Germany e.g. there are only few bonds with time to maturity beyond 10 years, whereas in the United Kingdom government bonds are spread along a maturity range up to 25 years.

[9] The members of the G7 group are United Kingdom, Germany, France, United States, Japan, Italy and Canada.

The exponential term $\exp[-t\, r(t)]$ in equation (1) is the relevant discount factor $d(t)$ for the time period t. Thus, the present value of the cash flow F_t is the product of its nominal value and its discount factor.

$$P = d(t)\ F_t \tag{2}$$

Forward rates indicate interest rates for a time period starting at a future point in time. Implicit forward rates are defined by the equality of the terminal wealth of a long-term and a sequence of short-term investments. The following example for a t_2-year strategy with continuous-time compounding illustrates this relationship.

$$t_2\, r(t_2) = t_1\ r(t_1) + (t_2 - t_1)\ f(t_1, t_2)\,, \tag{3}$$

where $r(t_2)$ is the continuous-time spot rate for t_2 years, $r(t_1)$ is the spot rate for a shorter time period t_1, and $f(t_1, t_2)$ is the implied continuous-time forward rate for the time period starting after t_1 years and ending at t_2 years. All spot and forward rates on both sides of the equation are known at the initiation of the strategies. The spot rate $r(t_1)$ then indicates the average rate over t_1 years whereas the instantaneous forward rate measures the marginal return for the period from t_1 to t_2. If the time period $t_2 - t_1$ is indefinitely small $f(t_1, t_2)$ is called the instantaneous forward rate.[10]

Equation (3) can be solved for the forward rate. In general, the continuously compounded forward rate from t_1 to t_2 is then given by

$$f(t_1, t_2) = \frac{t_2}{t_2 - t_1}\ r(t_2) - \frac{t_1}{t_2 - t_1}\ r(t_1) \tag{4}$$

The sequence of short-term investments on the right hand of equation (3) side can be extended. Assume we split the time period t_n in N intervals (t_n, t_{n+1}) of equal time length. The connection between spot and forward rates becomes

$$r(t_n) = \frac{1}{n}\ [r(t_1) + f(t_1, t_2) + f(t_2, t_3) + \ldots + f(t_{n-1}, t_n)]\,. \tag{5}$$

This formulation reveals that any spot rate can be considered as an average of the relevant forward rates.[11] The importance of the spot rate $r(t_1)$ in the bracket diminishes as the number of intervals increases or can be replaced by a forward rate for a period starting in an instant and lasting until t_1. In the limit the spot rate $r(t)$ equals the instantaneous forward rate $f(x)$ integrated from 0 to t.

$$r(t) = \frac{1}{t} \int_0^t f(x)\ \mathrm{d}\,x \tag{6}$$

[10] As the time period between t_1 and t_2 is infinitesimal only one time index is needed. Therefore the notation $f(t)$ will be used for the instantaneous forward rate.

[11] If it is assumed that the forward rates do not contain a term premium, forward rates can be interpreted as expected future spot rates. For a discussion of the expectation theory see e.g. Shiller (1990).

The instantaneous forward rate provides the interest rate of a future loan that is repaid an instant later. The forward rate between t_1 and t_2 is then defined as

$$f(t_1, t_2) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} f(x) \; \mathrm{d}\, x, \tag{7}$$

where $f(x)$ again denotes the instantaneous forward rate.

3.2 Bond Price and Bond Yield

A bond consists of a predetermined series of cash flows. For non-callable Swiss government bonds the cash flows correspond to the annual fixed coupon payments plus the repayment of the face value at maturity. The price of a straight bond equals the sum of the present value of these individual cash flows. C_t denotes the coupon payment and F_T the repayment of the face value at maturity T. For the sake of simplicity, the formulas (8)-(10) are shown for a bond with exactly T full years remaining until maturity. It is assumed that the coupon payment just occurred and the first cash flow is the coupon due in exactly one year. Consequently, no accrued interest has to be taken into account.

$$B = \sum_{t=1}^{T} \exp\left[-t\, r(t)\right] \; C_t + \exp\left[-T\, r(T)\right] \; F_T \tag{8}$$

Equation (8) describes the price of a bond price given the continuously compounded spot rates $r(t)$. Expressed alternatively, the price is equal to the sum of the cash flows multiplied by the discount factors $d(t)$ applicable to the date of the cash flow in t years.

$$B = \sum_{t=1}^{T} d(t) \; C_t + d(T) \; F_T \tag{9}$$

Given the price of the bond we can determine the yield to maturity y. The yield to maturity[12] is the constant discount rate that equates the present value of the future cash flows to the current price B. It is calculated as the solution of the equation

$$B = \sum_{t=1}^{T} \exp(-t\, y) \; C_t + \exp(-T\, y) \; F_T. \tag{10}$$

[12] Equivalently some textbooks use the term (gross) redemption yield.

Implicitly it is assumed that coupon payments during the remaining time to maturity can be reinvested at the yield to maturity. In practice, the reinvestment takes place at the prevailing spot rate and hence the usage of the yield to maturity may not be appropriate in certain circumstances.

The above bond pricing equations describe the correct value of a bond assuming that the time until the next coupon payment is exactly one year. If this is not the case, accrued interest must be deducted from the bond value to get the quoted bond price. Denote the time of the next coupon payment as t_C and the settlement day as t_0. Then the quoted bond price is given as

$$B^{quoted} = B - C\left[1 - (t_C - t_0)\right]. \tag{11}$$

3.3 Spot and Forward Rate Curve, Discount Function

The most common representation of interest rates over maturity is the spot rate curve. A spot rate $r(t)$ is equivalent to the yield to maturity of a zero coupon bond, i.e. the interest rate to discount a single payment in t years. Therefore the name zero coupon yield curve is often used instead.

Implied forward rates can be derived from the spot rate curve (equation 4). The spot rate $r(t)$ measures the average interest rate until t whereas the instantaneous forward rate $f(t)$ equals the marginal interest rate. Figure 1(a) illustrates the relationship between the spot rate curve and the implied instantaneous forward rate curve. The spot and forward rate curve start at the same point at $t = 0$. The forward rate curve is above (below) the spot rate if the spot rate is increasing (decreasing). The forward rate curve then crosses the spot rate curve at its maximum (minimum).[13]

Recall from equation (2) that any spot rate $r(t)$ can be transformed into a discount factor $d(t)$. A future cash flow multiplied by the relevant discount factor is equal to the present value of this cash flow. Discount factors for zero coupon bonds with different maturities can be combined to form the discount function. In order to exclude arbitrage, the discount function must be non-increasing with time to maturity. Figure 1(b) plots the discount function derived from the spot rate curve to the left.

4 The Nelson/Siegel Model and its Extension by Svensson

4.1 The Nelson/Siegel Model

The starting point for the Nelson and Siegel (1987) model is the formulation of the process for the instantaneous forward rate. The authors assume

[13] Shiller (1990) compares the situation to average (spot rate) and marginal (instantaneous forward rate) costs.

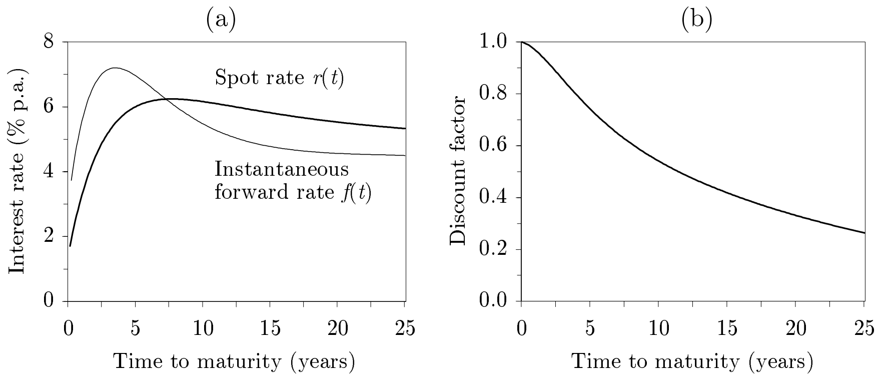

Figure 1: The relationship between (a) the spot rate, intantaneous forward rate and (b) the corresponding discount function.

that the instantaneous forward rates at any time t is given by the following parsimonious functional form.

$$f\,(t) = \beta_0 + \beta_1\ \exp\left(-t\,/\,\tau_1\right) + \beta_2\ \left(t\,/\,\tau_1\right)\ \exp\left(-t\,/\,\tau_1\right) \tag{12}$$

Forward rates are represented as a sequence of exponential terms. Nelson and Siegel argument that exponential functions are capable of capturing most shapes of the term structure. β_0 is a constant, the exponential term $\beta_1 \exp\left(-t\,/\,\tau_1\right)$ is monotonically decreasing (increasing) with time to maturity t if β_1 is positive (negative). The second exponential term $\beta_2\left(t\,/\,\tau_1\right)\exp\left(-t\,/\,\tau_1\right)$ produces a hump (trough) if β_2 is positive (negative). If the time to maturity converges to infinity both exponential functions become zero and the limiting value of equation (12) is β_0. If time to maturity approaches zero the exponential functions become 1, but the β_2 term drops out as it includes the fraction (t/τ_1). Hence the result is $\beta_0 + \beta_1$.

Section 3 showed that spot rates can be represented as an average of relevant forward rates. In continuous time this turned out to be the definite integral of the instantaneous forward rate with limits of integration of 0 and t, divided by t. Equation (13) follows by integrating equation (12) from 0 to t and dividing by t.

$$\begin{aligned} r\,(t) &= \beta_0 + \beta_1\left[\frac{1-\exp\left(-t\,/\,\tau_1\right)}{t\,/\,\tau_1}\right] \\ &\quad + \beta_2\left[\frac{1-\exp\left(-t\,/\,\tau_1\right)}{t\,/\,\tau_1} - \exp\left(-t\,/\,\tau_1\right)\right] \end{aligned} \tag{13}$$

Nelson and Siegel (1987) suggest that setting $\tau_1 = 1$, $\beta_0 = 1$ and $\beta_1 = -1$ allows best to explore the available shapes for the term structure.[14] As they

[14] Analogous to Nelson and Siegel (1987) Fig. 1: Yield curve shapes.

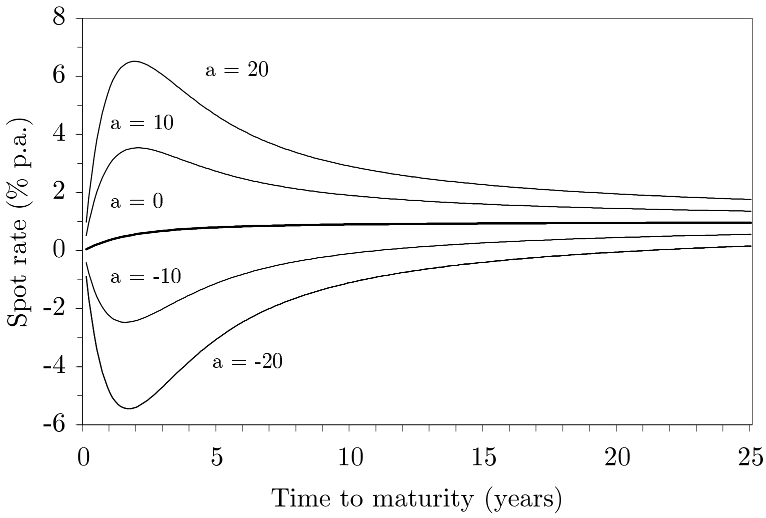

Figure 2: Setting $(\beta_0 + \beta_1) = 0$, $\beta_2 = -1$ and $\tau_1 = 1$ allows to illustrate the available shapes of the Nelson/Siegel model depending on a single parameter a that is varied from -10 to $+20$ with a step size of 10.

point out in the original paper, the expression for $r(t)$ then becomes

$$r(t) = 1 - (1-a)\, \frac{[1-\exp(-t)]}{t} - a\, \exp(-t)\,. \tag{14}$$

The shapes only depend on the single unknown parameter a. In figure 2 the parameter a takes the values from -20 to $+20$ with an increment of 10. All term structures converge to $\beta_0 = 1$. For $a = 20$ the term structure exhibits a hump whereas for $a = -20$ a trough results. For $a = 0$ the curve is monotonically increasing and concave.

4.2 Decomposition of the Nelson/Siegel Spot Rate Curve

Figure 3 illustrates the decomposition of the Nelson/Siegel spot rate curve into single exponential expressions. The figure uses a rather exceptional parameter constellation and shape of the term structure in order to illustrate the different time-dependent components. The decomposition of the spot rate into the three components is similar to the forward rate. β_0 represents the long-term interest rate that is approached in the limit. The β_1 term has the faster decay towards zero than the β_2 term and determines the short-term segment. The β_2 term converges to zero at $t = 0$ and $t = \infty$. Consequently, it has an impact on the medium-term segment of the term structure. The knowledge of the exact sensitivities of the term structure from the parameters will be used to set up boundaries for the estimation.

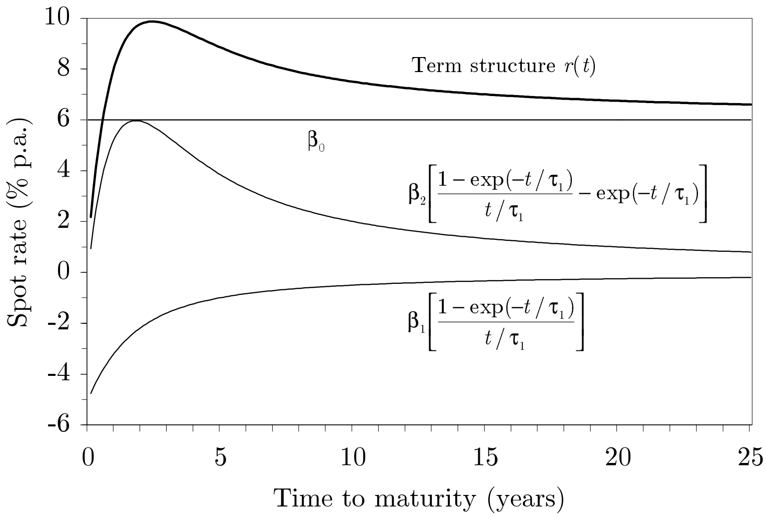

Figure 3: Decomposition of the Nelson-Siegel term structure into the long-term interest rate β_0 and the two exponential components. The bold solid line represents the resulting term structure of spot rates $r(t)$. The exemplary parameter values are $\beta_0 = 6$, $\beta_1 = -5$, $\beta_2 = 20$ and $\tau_1 = 1$.

The limiting values of the spot rate curve are equal to those of the forward rate curve. With longer time to maturity the spot rate curve approaches β_0. To avoid negative interest rates β_0 must be positive. If t gets small the limiting value for $r(t)$ is $(\beta_0 + \beta_1)$. Thus, also the sum $(\beta_0 + \beta_1)$ is required to be positive.

$$\lim_{t\to\infty} r(t) = \beta_0 \quad \text{and} \quad \lim_{t\to 0} r(t) = \beta_0 + \beta_1 \tag{15}$$

The parameter τ_1 is bounded to positive values that guarantee convergence to the long term value β_0. An increase in τ_1 shifts any hump or trough in the spot rate curve to the right or the decay to the long term rate is slower.

If the parameter β_1 is negative and its value in absolute terms is bigger than or equal to $|\beta_2|$ the term structure is monotonically increasing. If in addition $\beta_2 \geqslant 0$, then it is monotonically increasing and concave, i.e. the spot rate curve exhibits no change in the curvature (second derivative). To obtain a monotonic decay in the term structure the parameter β_1 is set to a positive value. Still the condition $|\beta_1| \geqslant |\beta_2|$ must hold. If the parameters β_1 and β_2 are both positive (negative) and $|\beta_1| < |\beta_2|$ this produces a hump (trough) that lies above (below) the long term interest rate β_0. However, if the sign of β_2 is changed the hump (trough) crosses the β_0-line. This corresponds to the situation of figure 3: A hump-shaped spot rate curve that crosses the long-term rate around a time to maturity close to one. In

Shape of the spot rate	β_0	β_1	β_2	τ_1	Condition
Increasing, concave	+	−	+	+	$\lvert\beta_1\rvert \geqslant \lvert\beta_2\rvert$
Increasing	+	−	−	+	$\lvert\beta_1\rvert \geqslant \lvert\beta_2\rvert$
Decreasing, convex	+	+	−	+	$\lvert\beta_1\rvert \geqslant \lvert\beta_2\rvert$
Decreasing	+	+	+	+	$\lvert\beta_1\rvert \geqslant \lvert\beta_2\rvert$
Hump, above β_0	+	+	+	+	$\lvert\beta_1\rvert < \lvert\beta_2\rvert$
Hump, crosses β_0	+	−	+	+	$\lvert\beta_1\rvert < \lvert\beta_2\rvert$
Trough, below β_0	+	−	−	+	$\lvert\beta_1\rvert < \lvert\beta_2\rvert$
Trough, crosses β_0	+	+	−	+	$\lvert\beta_1\rvert < \lvert\beta_2\rvert$

Table 1: The term structure shapes resulting from all possible parameter constellations.

general, the bigger β_2 compared to β_1 the more accentuated the hump or trough becomes.

Figure 5 shows the estimation of the term structure for the last day of the sample period, July 31, 1998. Because the circles indicate the yield to maturity of coupon bearing bonds they need not coincide with the spot rate curve. This effect becomes more apparent for long-term bonds that include numerous outstanding payments. Apart from the very short run the observed yields (circles) are well approximated by the fitted yields (small black squares).

4.3 The Extended Version of Svensson

Svensson (1994) augments the Nelson/Siegel approach by an additional exponential term. This term allows for further flexibility in form of a second possible hump or trough. As the overview of the literature in section 2 has shown, several investigations extend their analysis to the version by Svensson. This section briefly reviews the key differences between the two models. However, because of the limited number of available and liquid default-free bonds on the Swiss fixed income market, the empirical analysis in section 6 will restrict to the Nelson/Siegel model. Test calculations revealed that the overparametrization of the Svensson model for the Swiss market causes serious convergence problems.[15] Moreover, the empirical evidence from other countries demonstrates no significant improvement of the estimations with the Svensson extension. Schich (1997) explores the improvement of the fit when applying the Svensson approach to German government loans, bonds

[15] In the U.K. there are more than 50 regularly traded government bonds available for the estimation of the nominal spot rate curve and the Svensson model can even be augmented by further parameters to model the tax effects. For the estimation of the real rates Deacon and Derry (1994a) report that the Bank of England uses a relatively small sample of 13 index-linked gilts. As a consequence, they also estimate the parsimonious Nelson/Siegel model.

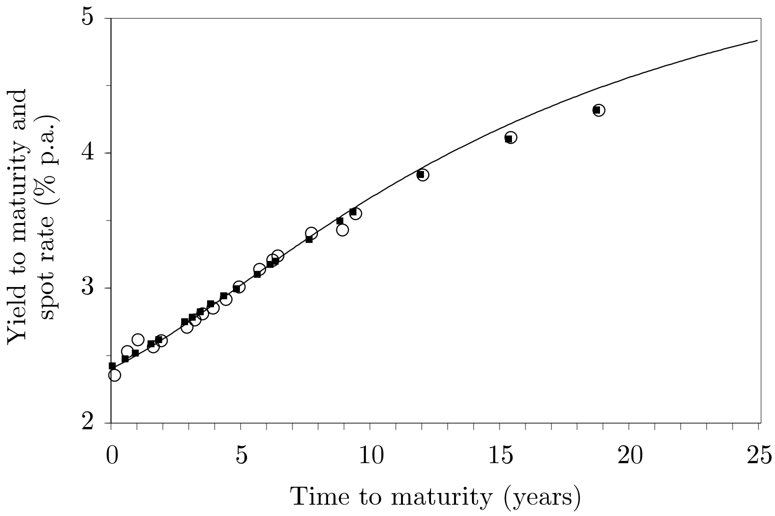

Figure 4: Estimation of the spot rate on July 31, 1998 using the Nelson and Siegel (1987) model. The circles indicate the yields to maturity of the individual Swiss government bonds, the small black squares the fitted yields, and the solid line the resulting spot rate curve.

and Treasury bills. He concludes that at least with monthly data for the investigated sample period from September 1972 to December 1996, the additional flexibility shows no gain in the form of substantially smaller yield errors in the objective function. The results are confirmed by Geyer and Mader (1999) and Ricart and Sicsic (1995).

Mathematically, two instead of one change in the second derivative of the spot rate curve are feasible with the Svensson extension.

$$\begin{aligned} f(t) &= \beta_0 + \beta_1 \exp(-t/\tau_1) + \beta_2 (t/\tau_1) \exp(-t/\tau_1) \\ &\quad + \beta_3 (t/\tau_2) \exp(-t/\tau_2) \end{aligned} \tag{16}$$

The forward rate representation chosen by Nelson/Siegel belongs to a class of functions called Laguerre functions (see chapter 9 in Press et al., 89). These functions are characterized by a polynomial times a decaying exponential term. The use of Laguerre functions is a well-known approximation procedure. In that light the Svensson approach follows as a natural extension of the Nelson/Siegel approach, including a third Laguerre term.

As in the case of the Nelson/Siegel model the spot rates can be derived by integration according to equation (6). The spot rates $r(t)$ are then given

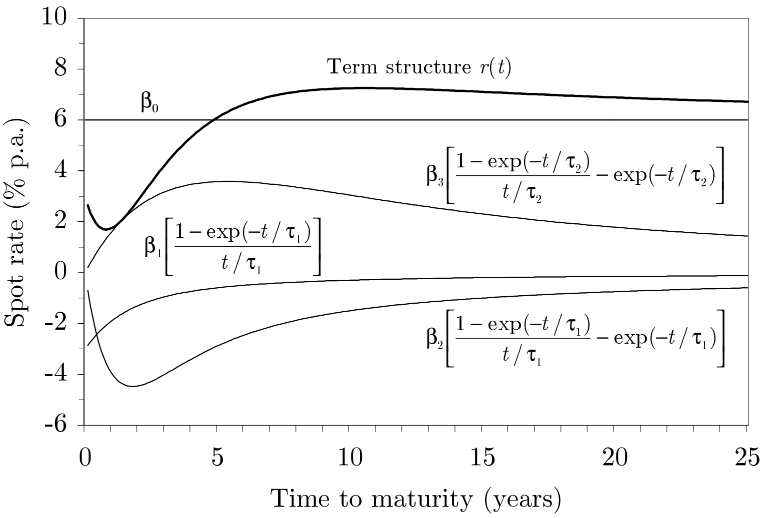

Figure 5: Decomposition of the Svensson spot rate curve into the long term interest rate β_0 and the three exponential terms. The parameter values are $\beta_0 = 6$, $\beta_1 = -3$, $\beta_2 = -15$, $\tau_1 = 1$, $\beta_3 = 12$ and $\tau_2 = 3$.

by

$$\begin{aligned} r(t) &= \beta_0 + \beta_1 \frac{1-\exp(-t/\tau_1)}{t/\tau_1} + \beta_2 \left[\frac{1-\exp(-1/\tau_1)}{t/\tau_1} - \exp(-t/\tau_1)\right] \\ &\quad + \beta_3 \left[\frac{1-\exp(-t/\tau_2)}{t/\tau_2} - \exp(-t/\tau_2)\right]. \end{aligned} \tag{17}$$

Note that the β_3 term is identical to the β_2 term with τ_1 replaced by τ_2. The two additional parameters β_3 and τ_2 explain the extended flexibility of the Svensson approach. Similar to τ_1, small values of the new parameter τ_2 correspond to rapid decay of the additional hump or trough towards the limiting value of β_0.

Figure 5 decomposes the Svensson spot rate curve, denoted by the solid bold line, into its components. The β_2 term causes the resulting spot rates to be hump-shaped in the short run, i.e. for a time to maturity of 0-3 years. The β_3 term itself has the maximum at a time to maturity of 5.4 years, but the combination with the two other exponential functions results in a hump of the spot rate curve with a maximum at 10.6 years. Beyond this maximum the term structure slowly converges to β_0. Over the whole maturity spectrum the term structure changes twice its curvature, from convex to concave, and again back to a convex decay towards β_0.

5 The Numerical Procedure

The first part of this section shows an efficient way to store the information of the observed market data in three matrices. Then the general steps of the estimation of the Nelson/Siegel method are discussed. The specific implementation for the use with Swiss government bonds is the topic of section 6.

Non-callable bonds are fully characterized by the coupon rate, face value and time to maturity. The two matrices **C** (for cash flows) and **T** (for times to payment) summarize this information. The dimension of the two matrices is determined by the number of bonds N and the maximum number of full years of the bond with the longest time to maturity.[16] The row index i numbers consecutively the bonds from 1 to N and each row represents a specific bond. The matrix **C** contains the future cash flows, namely the coupon payments C_{ij} and the repayment of the face value F_{iT}. The column index j runs from 1 to T, where 1 indicates the next coupon payment and T the maturity. The elements of the matrix **T** describe the remaining times to payment corresponding the cash flows $\mathbf{C}_{ij}$.[17] Both matrices are padded to the right with zeros,

$$\mathbf{C} = \begin{bmatrix} F_{1T} & 0 & 0 & \cdots & 0 \\ C_{21} & C_{22} & C_{2T} + F_{2T} & \cdots & 0 \\ \vdots & & & & \vdots \\ C_{N1} & C_{N2} & C_{N3} & \cdots & C_{NT} + F_{NT} \end{bmatrix} \tag{18}$$

and

$$\mathbf{T} = \begin{bmatrix} T_{1T} & 0 & 0 & \cdots & 0 \\ t_{21} & t_{22} & T_{2T} & \cdots & 0 \\ \vdots & & & & \vdots \\ t_{N1} & t_{N2} & t_{N3} & \cdots & T_{NT} \end{bmatrix}. \tag{19}$$

The bond prices are stacked to a vector.

$$\mathbf{b} = \begin{bmatrix} B_1 & B_2 & \cdots & B_N \end{bmatrix} \tag{20}$$

Figure 6 summarizes the steps of the estimation process for the Nelson/Siegel model.[18] The left branch (a)-(c) returns the yields to maturity from the market data. First, accrued interest is added to the quoted prices of Swiss government bonds. We then insert the adjusted bond price and solve for the yield to maturity y in the pricing formula

$$B = \sum_{t=1}^{T} \exp(-t\,y)\; C_t + \exp(-T\,y)\; F_T. \tag{21}$$

[16] All Swiss government bonds pay annual coupons.

[17] To simplify the notation a subscript denoting the settlement date is dropped.

[18] Exactly the same sequence of calculations can be applied to the Svensson approach.

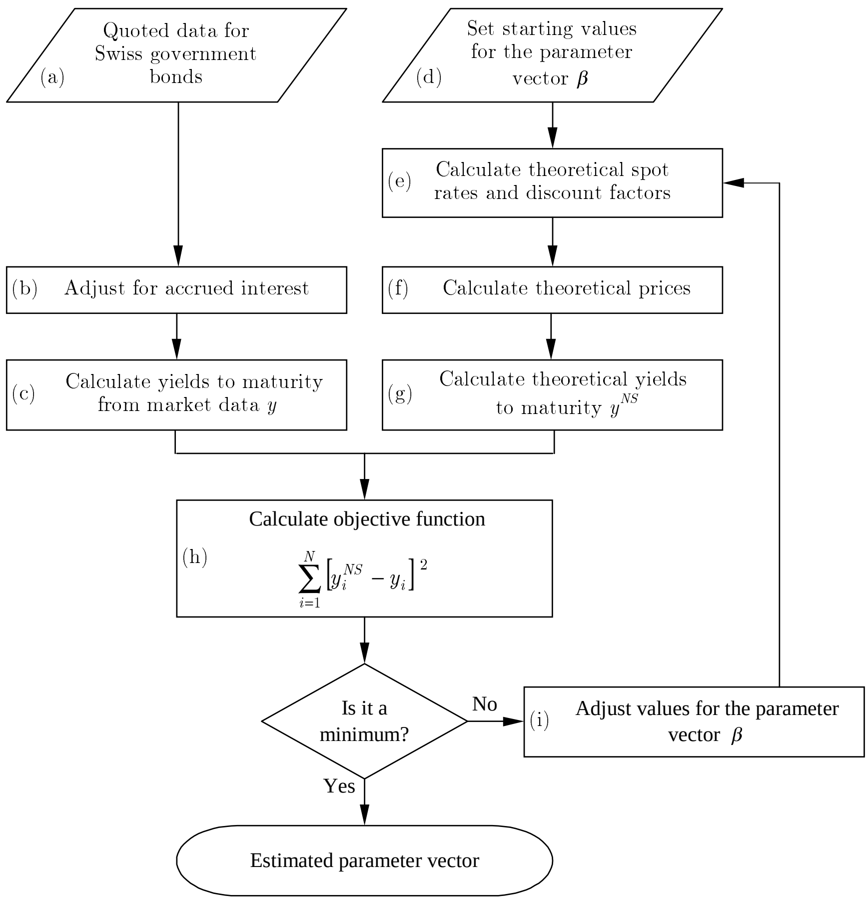

Figure 6: The flowchart illustrates the sequence of calculations necessary to estimate the parameters of the Nelson/Siegel model from observed bond prices.

There is no analytical solution and the equation must be solved numerically. The applied method is Newton-Raphson (see e.g. Hamilton, 1992).[19] Thus, out of the vector of quoted bond prices and, together with the matrices **C** and **T** representing the characteristics of these bonds, we calculate a new vector of yields to maturity observed on the market.

The right branch (d)-(g) in the flowchart begins by initializing the parameter vector β. In case of the Nelson/Siegel model the vector contains the following four elements.

$$\beta = \begin{bmatrix} \beta_0 & \beta_1 & \beta_2 & \tau_1 \end{bmatrix} \tag{22}$$

[19] At first sight the non-linear equation with the order equal to the number of coupon payments appears complex. However, as Svensson (1994) points out, the equation has only one real root and is thus easy to solve numerically.

The aim is to find a parameter vector that reflects the observed market yields to maturity as close as possible.[20] Inserting the estimated parameters into the formulation of the Nelson/Siegel spot rate curve (equation 13) returns the theoretical spot rates. These spot rates are transformed to discount rates and used in

$$B = \sum_{t=1}^{T} d(t)\ C_t + d(T)\ F_T \tag{23}$$

to get the corresponding theoretical bond prices.[21] Applying again the Newton-Raphson procedure equation (21) can be solved for the theoretical bond yields y^{NS}.

At point (g) on the flowchart, where the two branches meet, the sum of squared differences between the market yields and the theoretical yields to maturity of the N bonds is calculated. The objective function is minimized to get the appropriate estimates for β, i.e. the loop (i) and then (e)-(h) on the right hand side of the flowchart is repeated until the objective function reaches a minimum.

$$\min \sum_{i=1}^{N} \left(y_i^{NS} - y_i\right)^2 \tag{24}$$

Alternatively, the bond prices themselves could be approximated and price errors could be minimized. However, as Deacon and Derry (1994a) mention, minimizing yields implicitly gives greater weight to bonds with maturities up to about ten years and thus improves the fit of the curve at shorter maturities.[22] From the perspective of monetary policy or interest rate risk management this is desirable as this covers the interesting time horizon.

6 Implementation of the Nelson/Siegel Model for Switzerland

6.1 The Data

The Nelson and Siegel (1987) model is applied to Swiss government bonds from January 28, 1994 to July 31, 1998. The daily observations during

[20] Deacon and Derry (1994a) argue that the par yield curve was chosen as the resulting forward rate curve is more robust than if it is derived from a discount function.

[21] In matrix notation, a matrix of discount factors corresponding to the times to payment in **T** is calculated. Then the bond prices follow directly as the diagonal elements of **C** times the transpose of the matrix of discount factors.

[22] Ricart and Sicsic (1995) weight the pricing errors with the duration in order to mitigate this problem.

this data period amount to a total of 1176 observations.[23] Callable bonds are excluded from the data set as the call features may heavily distort the bond price. Furthermore, only bonds with a remaining time to maturity of more than 3 months are included in order to avoid market disturbances towards the end of the life of the bonds.[24] Instead, the 1-month and 1-year Euromarket rates are added to the data set. To include other default-free instruments than government bonds is a commonly used strategy to improve the fit at the short end of the term structure (see e.g. Svensson, 1994, or Heller, 1997).[25] At the beginning of the data period, the estimations are based on 13 government bonds and at the end a maximum of 18 bonds plus the two Euromarket rates. The coupon rates range from 3.5% for the bond maturing in August 2010 up to 7% for the July 2001 bond. Table 2 lists the bonds (and the two Euromarket rates) of the sample. For bonds that are newly issued during the data period, the second column of the table shows when the series begins. Initially, there are only the two Euromarket rates in the time segment from 0 to 6 years. Over the analyzed period of $4\frac{1}{2}$ years, there is a steady increase in the number of bonds, and the bonds get more and more equally spread over a range from roughly 2 to 19 years. Figure 4 at the end of section 4.1 represents the typical pattern at the end of the sample period. Apart from the bond characteristics the table shows the minima and maxima for the prices, yields and the daily changes in the yields. The final two columns contain the mean and standard deviation of the changes in the yield. These standard deviations are considerably higher for the shortest maturities.

6.2 Calibration

The iteration to estimate theoretical bond yield starts with initializing the parameter vector β. To ensure convergence towards the true minimum of the objective function these starting values need to be chosen carefully. For the first loop at the beginning of the data set, on January 28, 1994, the knowledge of the limiting values and sensitivities of the Nelson/Siegel model are applied. Because β_0 represents the long-term interest rate, it takes the yield of the bond with the longest time to maturity. The limiting value of the spot rate if t gets small is $(\beta_0 + \beta_1)$. If we set this sum equal to the yield of the shortest maturity, the 1-month Euromarket rate, and replace β_0 by the yield of the longest maturity it follows naturally to set β_1 equal to the difference between the 3-month Euromarket rate minus the yield

[23] Source: Datastream.

[24] This agrees with Schich (1997) for the Deutsche Bundesbank.

[25] Nelson and Siegel (1987) warn against the use of very short maturities like the overnight rate or Treasury bills with the shortest term of 3 and 10 days: "Yields are consistently higher, presumably because of relatively large transaction costs over a short term to maturity."

Euromarket rate/bond				Bond price		Yield (% p.a.)		Return (% p.a.)			
Maturity	First price[a]	Coupon	Obs.	Min.	Max.	Min.	Max.	Min.	Max.	Mean	Std. Dev.
1-month		0.00	1176			0.729	4.491	-0.568	0.395	-0.002	0.090
1-year		0.00	1176			1.340	4.704	-0.359	0.375	-0.001	0.078
10-Mar-99	18-Mar-94	4.00	1140	96.107	108.536	1.044	5.304	-0.551	0.419	-0.002	0.055
10-Jun-00		4.50	1176	95.849	112.089	1.294	5.411	-0.356	0.340	-0.002	0.047
11-Mar-00		5.00	1176	99.556	113.890	1.273	5.388	-0.349	0.420	-0.002	0.048
07-Oct-01	26-Sep-94	5.50	1004	100.110	117.469	1.618	5.500	-0.209	0.212	-0.003	0.036
09-Jul-01		7.00	1176	109.632	125.442	1.580	5.525	-0.254	0.302	-0.002	0.040
05-Feb-02		6.50	1176	107.482	123.642	1.718	5.521	-0.211	0.225	-0.002	0.037
08-Jul-02		4.50	1176	94.538	114.399	1.719	5.542	-0.239	0.217	-0.001	0.041
07-Jan-03		6.25	1176	105.600	124.103	1.848	5.606	-0.236	0.310	-0.001	0.042
11-Jun-03		6.75	1176	109.416	128.723	1.932	5.628	-0.348	0.445	-0.001	0.040
10-Apr-04		6.50	1176	107.911	130.688	2.096	5.634	-0.332	0.443	-0.001	0.039
07-Oct-04		4.50	1176	92.035	115.615	2.209	5.620	-0.195	0.242	-0.001	0.040
06-Jan-05	28-Nov-94	5.50	959	101.150	122.135	2.275	5.355	-0.247	0.187	-0.003	0.034
08-Apr-06		4.50	1176	91.413	118.759	2.458	5.644	-0.203	0.274	-0.001	0.039
10-Jun-07	30-May-96	4.50	566	99.424	118.382	2.626	4.623	-0.123	0.140	-0.003	0.033
08-Jan-08	27-Nov-95	4.25	699	97.138	113.800	2.711	4.604	-0.133	0.195	-0.001	0.035
07-Aug-10	28-Aug-97	3.50	264	96.500	107.782	2.923	3.923	-0.130	0.118	-0.001	0.031
06-Jan-14		4.25	1176	83.603	112.022	3.320	5.951	-0.184	0.299	-0.001	0.040
05-Jun-17	26-May-97	4.25	309	100.170	13.414	3.511	4.287	-0.125	0.129	-0.002	0.027

[a] Where no date is indicated the full sample is available.

Table 2: Summary statistics of 1176 observations from January 28, 1994 to July 31, 1998. Besides the 18 non-callable Swiss government bonds two Euromarket rates are included at the top of the table.

of the bond with the maximum time to maturity. As the term structure at the beginning of 1994 is increasing the remaining two parameters are set to β_2 to -1 and $\tau_1 = 1$. These values turned out to work well for the investigated data period. After the first initialization, the estimated parameters of the previous day are taken as the new starting values for the subsequent estimation.

The sum of squared deviations of the theoretical bond yields from the observed ones described by equation (24) can be formulated as a log likelihood function.

$$\log L = -N \ \log\left(2\pi\sigma^2\right) - 0.5 \sum_{i=1}^{N} \frac{\left(y_i^{NS} - y_i\right)^2}{\sigma^2} \tag{25}$$

The variance σ^2 is calculated as the sum of squared deviations divided by the number of observations N. This log likelihood function is maximized by changing the parameter vector β that determines the theoretical bond yields.[26] The log likelihood function described by equation (25) is constrained by non-negativity conditions for the parameters β_0 and τ_1. In addition, the following boundaries proved to be successful for the considered sample.

- Besides being never negative β_0 is not allowed to deviate more than 3 percentage points from the bond yield with the longest maturity.
- β_1 has to lie in the range of ± 3 percentage points from the difference between the 3-month Euromarket rate minus the bond yield with the longest maturity.
- The lower bound for β_2 is set to -10 and the upper bound to 20.
- The boundaries for τ_1 are 0.05 and 20.

The algorithm used to solve this nonlinear programming problem is the Sequential Quadratic Programming method contained in the Matlab Optimization Toolbox.[27] All restrictions and boundaries were never binding for the whole sample, however they are crucial to achieve plausible solutions.

6.3 Empirical Results

Figure 7 shows the evolution of the parameter values from January 24, 1994 to July 31, 1998. The parameter β_0 reveals that the long-term interest rate

[26] The termination conditions are set as follows: The objective function requires a precision at the solution of 1*e*-4 and the worst case constraint violation that is acceptable is 1*e*-7. A maximum number of iterations of 300 was never exceeded.

[27] See The MathWorks Inc. (1996) User's Guide for the Optimization Toolbox, chapter 2: Introduction to Algorithms, and especially pages 2-22, Constrained Optimization.

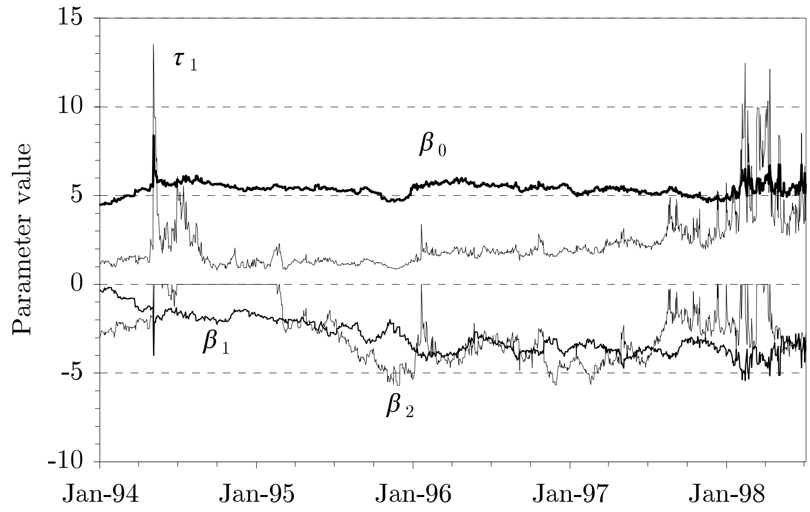

Figure 7: Daily parameter values of the Nelson-Siegel model for the period from January 1994 to July 1998.

changed only slightly over the sample period. However, recall from section 4 that β_0 determines the limiting value and for finite times to maturity the combination with the other parameters matters. The 20-year interest rate for instance decreases from a maximum of 5.780% on September 9, 1994 to 4.057% at the end of the sample.

Short-term interest rates dropped during the observed data period as it can be seen from the sum of β_0 and β_1. With short interruptions at the end of April 1994 until mid of May 1994 and in June 1994 (from June 10 to June 26) the short term interest rate was above 4% until September 12, 1994. Then with the exception of a last period of three days in December 1994 (7-9 December) the short rate never exceeded 4% again. The short-term rate dropped even below 1% at end of May 1997 and from the mid February 1998 until mid of March 1998.

The β_1 parameter constantly decreases and remains negative during the whole period. It describes the short-term component and a negative value indicates an increasing term structure. At the beginning the term structure is basically flat with a slight trough at the short end and hence the absolute value of β_1 is close to zero and smaller than $|\beta_2|$. Until September 1994 the long term rates increase and the slope of the term structure becomes positive. Despite occasional troughs (below β_0) in the very short run the increasing shape persists over the remaining sample period. The troughs occur any time when in absolute terms the value of β_1 is smaller than β_2.

It is readily apparent, that β_2 and τ_1 show much more pronounced changes over time. Moreover, it can be seen that during several periods β_2 takes the value zero. $\beta_2 = 0$ together with a negative β_1 coefficient signifies a monotonically increasing and concave term structure. As it can be seen from figure 7 the term structure is concave most of the time from June 94 to March 95, on February 16, 1996, and several times during the period from January to May 1998. Hump shapes that require big positive values of β_2 do not occur in the sample.

A change to a value of zero for β_2 is usually accompanied by an increase in τ_1. An increase in τ_1 corresponds to a shift of any curvature in the term structure to the right. If β_2 gets zero the hump in the short run disappears and hence the slightly bent curve has no longer a pronounced curvature at the short end. This explains the inverse relationship between the two parameters. But it should be stressed that a high value of τ_1 does not imply an extraordinary shape of the term structure, as e.g. on June 1, 1994 with $\tau_1 = 13.525$. Big (Small) values simply enable to fit the curvature well at long (small) maturities.

6.4 Robustness Tests

Four measures are analyzed to check the accuracy of the estimation results.

- The terminal value of the log likelihood function.
- The absolute and relative average yield error.
- The spread between the highest and smallest deviation
- Root mean squared yield errors

In figure 8 the value of the log likelihood function evaluated at the solution decreases over the observation period. To some extent this follows naturally from the increase of the number of available bonds. At the beginning of the data period only the two Euromarket rates form the short segment of the term structure whereas towards the end the observations are spread over the whole maturity range and an appropriate fit becomes much more difficult. However, it is apparent that the volatility of the log likelihood function increases at the end of 1997 until the beginning of 1998.

Figures 9 and 10 make an interesting comparison between the average absolute yield error $\frac{1}{N}\sum_{i=1}^{N}\left(y_i^{NS} - y_i\right)$ and the average relative yield error $\frac{1}{N}\sum_{i=1}^{N}\left(y_i^{NS} - y_i\right)/y_i$. The average absolute yield error is measured in basis points (= 0.01 percentage points) and the average relative yield errors in percent. The average absolute error does not deviate substantially from zero. During the major part of the sample the average absolute error is negative, meaning that the observed yields are slightly underestimated by the model.

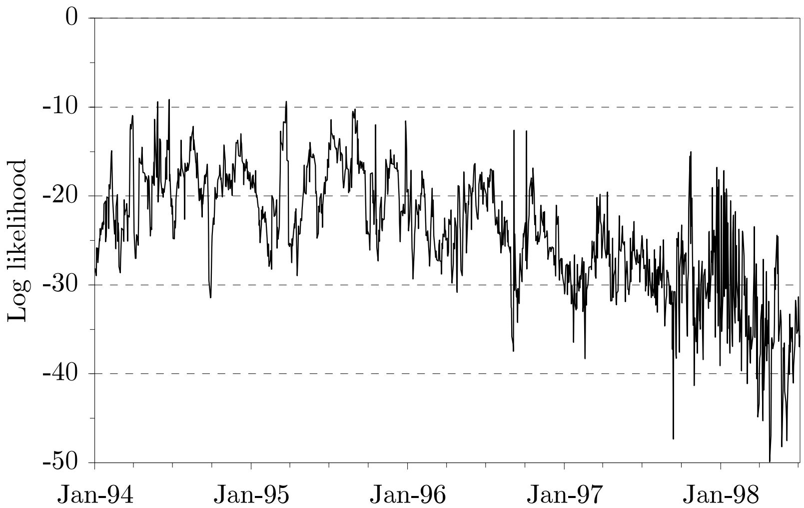

Figure 8: The log likelihood function (see equation 25 in the text) daily evaluated at the solution.

On the other hand the average relative yield errors amount to values up to 0.491% on February 20, 1998. The reason is that the interest rate level at the beginning of 1998 is extremely low, especially at the short end, and hence small absolute deviations amount to considerable percentage deviations. On February 20, 1998, the 1-month Euromarket rate is at 0.792% and the theoretical spot rate based on the Nelson/Siegel estimations is 0.141 percentage points, i.e. 17.848% higher. On the contrary the model value was 0.315 percentage points lower than the observed 1-year Euromarket rate of 0.918%, a difference $y^{NS} - y$ of -21.768%. The individual bond yields are overestimated up to 4.763% (the 4.5% July 2002 bond) and the biggest deviations concern mainly short-term bonds. On average a positive deviation from the market yields results, but as interest rates are at a low level these deviations are not substantial in absolute terms.

Figure 11 illustrates the root mean squared error $\sqrt{1/n \sum_{i=1}^{N} \left(y_i^{NS} - y_i\right)^2}$ and the absolute yield error spread, defined as the difference between the biggest positive deviation minus the biggest negative deviation

$$\left|\max\left(y_i^{NS} - y_i\right) - \min\left(y_i^{NS} - y_i\right)\right|. \tag{26}$$

Despite the increase in the number of bonds, the yield error spread does not increase over time. Both measures reveal similar characteristics and exhibit again an augmented volatility at the end of 1997 until the beginning of 1998. Big yield error spreads are usually due to a single bond yield observation, like on October 2, 1996 where the yield for the bond maturing in July 2002

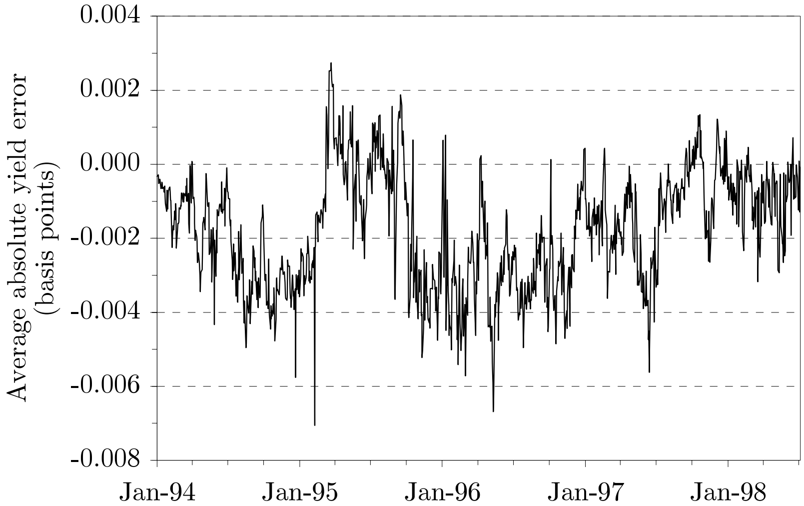

Figure 9: Average absolute deviations of the estimated yields from the observed Swiss government bond yields.

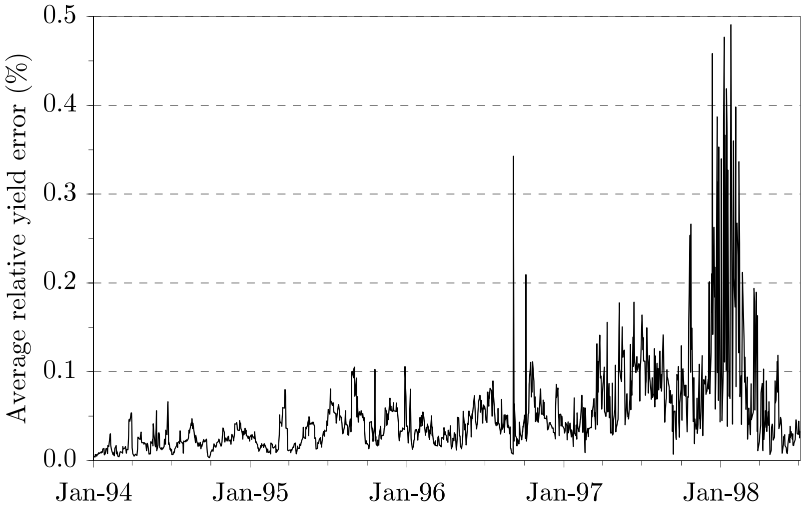

Figure 10: Average relative deviations of the estimated bond yields from the observed yields.

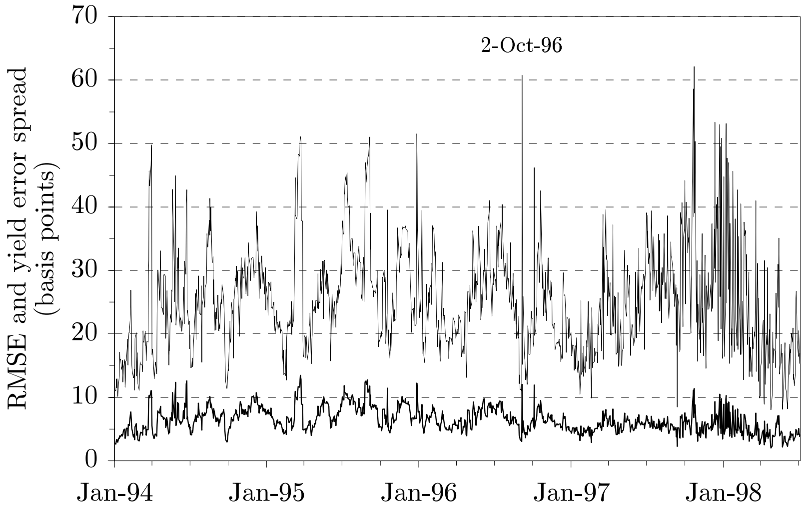

Figure 11: The root mean squared errors and the spread between the maximum positive and negative absolute deviation.

is overestimated by 0.394 percentage points, probably due to illiquid trading on that day.

7 Conclusions

The parsimonious Nelson and Siegel (1987)model and its extension by Svensson (1994) are widely used among central banks. This paper calibrates the Nelson/Siegel approach to Swiss market. The limited number of noncallable Swiss government bonds prevents the application of the Svensson model. After an introduction of the fundamental relationships on the fixed-income market, the sensitivities and the interpretation of the parameters in the Nelson/Siegel model are addressed in detail. The Nelson/Siegel model is compared to the extension by Svensson and the general estimation process is explained. The estimation is implemented as a constrained optimization problem and parametrized for the Swiss market. The estimation results suggest the following conclusions:

- The objective function to be minimized is the sum of squared deviations of the estimated yields from the observed market yields. The knowledge of the limiting values and the economic interpretation is needed to constrain the parameters so that convergence to accurate term structures on all 1176 observation days is guaranteed.
- The limiting long-term interest rate β_0 is stable over time and ex-

trapolations beyond the maturity range of the available bonds deliver plausible results. Considerable changes in the parameter values β_2 and τ_1 do not translate into big changes in the shape of the term structure. The average bond yield errors in absolute and relative terms are close to zero and also the root mean squared errors are small and quite stable over time and confirm that the estimated term structures are smooth and concise.

- The estimations provide smooth term structures. Single outliers in the data set do not affect the estimation results seriously. However, in the very short run, the lack of sufficient traded government bonds in Switzerland creates implausible curvatures. Consequently, the use for pricing short-term derivatives on long-term interest rate products is problematic. For monetary policy analysis on the other hand, where there is less need for a precise fitting of local anomalies than when pricing financial instruments, the Nelson/Siegel model is very attractive.

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